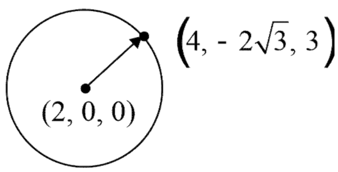
Solutions to JEE Main (2026) Sample Paper

PHYSICS

SECTION – 1

1.(B) Point source is placed at (2, 0, 0) and wave velocity is perpendicular to wavefront so $v = c\left(\frac{2\hat{i} - 2\sqrt{3}\hat{i} + 3\hat{k}}{5}\right)$



2.(C) Dimension of β : same as energy and α is dimensionless, so dimensions of $\frac{\beta}{\alpha}$ are that of energy.

3.(C) $30 - f = ma$ ……….(1)

$30R + fR = \frac{mR^2}{2}\frac{a}{R}$

$30 + f = \frac{ma}{2}$ ……….(2)

Using (1) & (2)

$60 = \frac{3}{2}ma \Rightarrow a = \frac{120}{30} = 4\ m/s^2$

$v^2 = u^2 + 2as = 0 + 2(4)(5) = 40$

$v = \sqrt{40}\ m/s$

4.(D) $H_2 = 32.0\,cm$

$H_1 = 22.0\ cm$

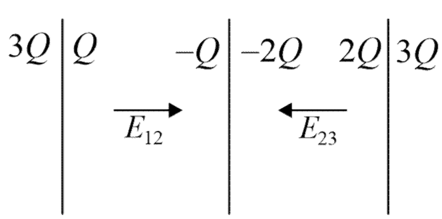
$\frac{\lambda}{2} = H_2 - H_1 = 32.0 - 22.0 = 10\ cm$

$\lambda = 20\ cm = 1/5\ (m)$

$v = f\lambda$

$v = 1615 \times \frac{1}{5} = 323\ m/s$

5.(A)

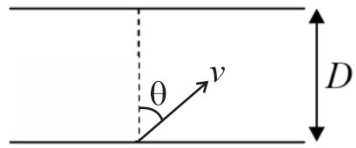


$E_{12} = \frac{Q}{A\varepsilon_0}$ $\quad E_{23} = \frac{2Q}{A\varepsilon_0}$

$\frac{E_{12}}{E_{23}} = \frac{1}{2}$

6.(C) $t_{\min} = \frac{2}{10}$

$t = \frac{4}{10} = \frac{D}{v\cos\theta}$



$\cos\theta = \frac{10D}{4v} = \frac{1}{2}$

$\theta = 60°$

Drift $= \frac{4}{10}\left(5 + \frac{10\sqrt{3}}{2}\right) = 2\left(\sqrt{3}+1\right)km$

7.(D) As cell is connected,

$E = \frac{v}{d}$; remains same

8.(D) Energy $10\times 50\times 3600 = 20\times t$

$t = 500\times 180 = 90000\,s$

9.(C) $\sin(wt) = \frac{x}{A}$, $\cos(wt) = \frac{y}{2A}$

$\sin^2(wt) + \cos^2(wt) = 1$

$\frac{x^2}{A^2} + \frac{y^2}{4A^2} = 1$

Path is elliptical with time period $\frac{2\pi}{w}$

10.(D) $M = \frac{\left(B-\mu_0 H\right)}{\mu_0}$

$= \left(\mu_r - 1\right)H = 700\times 2000\times 5 = 7\times 10^6\,\frac{A}{m}$

11.(C) Current through $500\,\Omega = \frac{20}{500} = \frac{1}{25}$

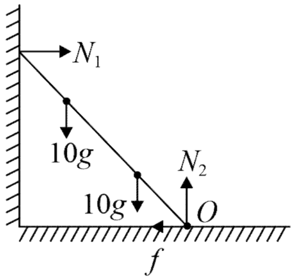
Current through load $= \frac{1}{25}\times\frac{2}{3} = \frac{8}{R_L}$

$R_L = \frac{25\times 3\times 8}{2} = 300\,\Omega$

12.(C) $\Delta Q = W = nc\Delta T$

So if $\Delta T \to 0$; C is infinite

13.(C)



$N_1 = f$
$N_2 = 20g$

Balance torque about O

$$N_1 \times 15 \sin 37° = 100 \times 10 \times \cos 37° + 100 \times 5 \times \cos 37°$$

$$N_1 = \frac{400}{3} = 133.3\,N$$

14.(D) $$B_{center} = \frac{\mu_0 i}{2R} + \frac{\mu_0 i R^2}{2\left(R^2 + 4R^2\right)^{3/2}} = \frac{\mu_0 i}{2R}\left[1 + \frac{1}{5^{3/2}}\right]$$

$$B_{\text{at midpoint}} = \frac{\mu_0 i R^2}{2\left(R^2 + R^2\right)^{3/2}} \times 2 = \frac{\mu_0 i}{2R}\left[\frac{1}{2^{1/2}}\right]$$

15.(A) $$E_1 = \frac{hc}{\lambda_1} = (13.6\,eV)\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 13.6\,eV\left[\frac{3}{4}\right]$$

$$E_2 = \frac{hc}{\lambda_2} = (13.6\,eV)\left[\frac{1}{2^2}\right] \cdot 9 \quad ; \quad \frac{\lambda_1}{\lambda_2} = \frac{9}{4} \times \frac{4}{3} = \frac{3}{1}$$

16.(B) Using energy conservation

$$-\frac{GM_e m}{2\mathrm{R_e}} + \frac{1}{2} m\left(\sqrt{\frac{5GM_e}{2\mathrm{R_e}}}\right)^2 = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{3GM_e}{2\mathrm{R_e}}}$$

17.(C) $$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{V} - \frac{1.5}{20} = \frac{1 - 1.5}{-18} \quad ; \quad \frac{1}{V} = \frac{1}{36} - \frac{3}{40} \quad = \frac{40 - 30 \times 36}{36 \times 40}$$

$$v = \frac{-36 \times 40}{68} = -21.18\,cm$$

18.(C)

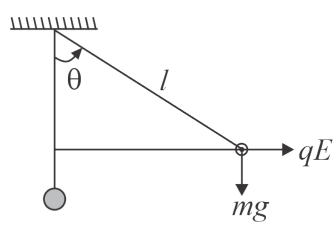
19.(B) By WET

$$qEl \sin\theta = mgl(1 - \cos\theta)$$

$$\frac{\sin\theta}{\sqrt{3}} = 1 - \cos\theta$$

$$\frac{\tan\theta}{2} = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{3} \text{ or } 60°$$

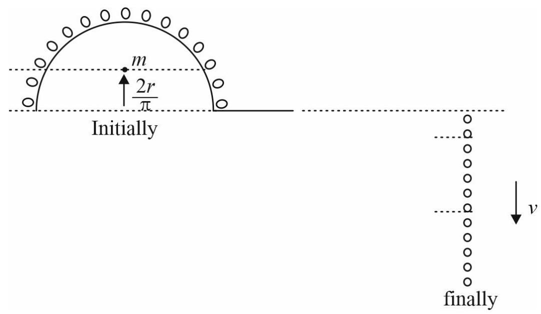


20. (C) Energy conservation

$$mg\frac{2r}{\pi}=\frac{1}{2}mv^2-mg\frac{\pi r}{2}$$

$$\frac{v^2}{2}=\frac{2gr}{\pi}+\frac{\pi gr}{2}$$

$$v=\sqrt{\frac{4gr}{\pi}+\pi gr}=\sqrt{2gr\left[\frac{2}{\pi}+\frac{\pi}{2}\right]}$$



SECTION – 2

1.(9) $F=\left[\left(P_0+\rho_1 gh_1\right)-\left(P_0+\rho_2 gh_2\right)\right]A=\left(\rho_1 h_1-\rho_2 h_2\right)gA$

$$=\frac{(2.4-1.5)\times 10^3\times 10\times 10}{10^4}=9$$

2.(30) $F=yA\frac{\Delta l}{l}=YA\alpha\Delta T$

$$\Delta T=\frac{F}{YA\alpha}=\frac{21600}{1.5\times 10^{11}\times 4\times 10^{-4}\times 1.2\times 10^{-5}}=\frac{21600}{7.2\times 10^2}=30$$

3.(8) $\phi=\vec{B}\cdot\vec{A}$

$\phi=BA\cos\theta$

A= Area of triangle $=\frac{\sqrt{3}a^2}{4}$; $\phi=B_0 e^{-\lambda t}\frac{\sqrt{3}a^2}{4}\times(1)$

emf $=\frac{-d\phi}{dt}=\frac{B_0\lambda e^{-\lambda t}\sqrt{3}a^2}{4}$; $I=\frac{emf}{3R}=\frac{B_0\lambda e^{-\lambda t}\sqrt{3}a^2}{4\times 3R}=\frac{B_0 e^{-\frac{t}{2}}a^2}{2\times 4\sqrt{3}R}$

$x=8$

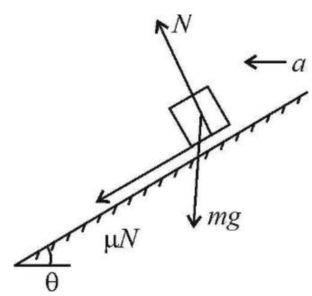
4.(20) From free body diagram in vertical direction we can say

$N\cos\theta=mg+\mu N\sin\theta$

$$N=\frac{mg}{\cos\theta-\mu\sin\theta}$$

So, $a=\frac{g\sin\theta}{\cos\theta-\mu\sin\theta}+\frac{\mu g\cos\theta}{\cos\theta-\mu\sin\theta}$

$$=g\left[\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\right]=20\,m/\sec^2$$



5.(42) In second half time

$$V_2=\frac{60\times 30\times 2}{60+30}=40\,m/s$$

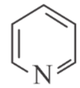
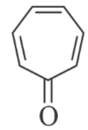
In entire journey

$$v=\frac{44+40}{2}=42\,m/s$$

CHEMISTRY

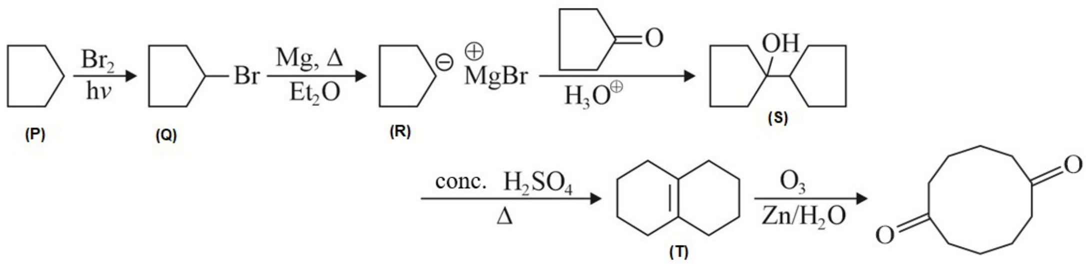
SECTION – 1

1.(D)

Thiophene  ; Pyridine  ; Tropone 

are non-benzenoid aromatic compounds.

2.(C)



3.(C) Correct order of melting point is:
B > Al > Tl > In > Ga

4.(B) Solubility of CO_2 in water is more than that of argon, $\therefore$ K_H value is lesser for CO_2.

5.(B) $E \propto \frac{Z^2}{n^2}$; $KE \propto \frac{Z^2}{n^2}$; $v \propto \frac{Z}{n}$; $r \propto \frac{n^2}{Z}$

$$\frac{(KE)_2}{(KE)_1} = \frac{\frac{1}{4}}{1} = \frac{1}{4} \; ; \; \frac{(TE)_2}{(TE)_4} = \frac{\frac{1}{4}}{\frac{1}{16}} = \frac{16}{4} = 4$$

$$\frac{(v)_3}{(v)_1} = \frac{\frac{1}{3}}{1} = \frac{1}{3} \; ; \; \frac{r_3}{r_1} = \frac{3^2}{1^2} = 9$$

6.(D) In isothermal process, temperature remains constant, while, in adiabatic expansion, temperature decreases as work is done at the cost of internal energy. T_{final} of reversible adiabatic expansion is least because more work is done in reversible than irreversible adiabatic expansion.

$$\Rightarrow T_a = T_b > T_d > T_c$$

7.(C) $y_X = \frac{P_X^o \chi_X}{P_X^o \chi_X + P_Y^o \chi_Y} = \frac{100}{100+50} = \frac{2}{3}$

8.(C)

$$MnO_2 + 4HBr \xrightarrow{\Delta} MnBr_2 + Br_2 + 2H_2O$$

$\frac{1}{87}$ mol $\quad \frac{1}{81}$ mol $\quad$ – $\quad$ – $\quad$ –

(L.R.)

$$g_{Br_2} = \frac{1}{4} \times \frac{1}{81} \times 160 = 0.494\,gm$$

9.(B) **(A)** $[Ag^+] = \sqrt{1.8\times10^{-10}} = 1.34\times10^{-5}\,M$

(B) $[Ag^+] = 2\times\sqrt[3]{\dfrac{1.1\times10^{-12}}{4}} = 1.3\times10^{-4}\,M$

(C) $[Ag^+] = 3\times\sqrt[4]{\dfrac{1.8\times10^{-18}}{27}} = 4.8\times10^{-5}\,M$

(D) $[Ag^+] = 2\times\sqrt[3]{\dfrac{6.0\times10^{-51}}{4}} = 2.3\times10^{-17}\,M$

10.(C)

Molecule	Hybridization	Structure	Dipole moment	Lone pair
PCl_3	sp^3	Pyramidal	Non zero	1
BrF_3	sp^3d	Bent T-shape (planar)	Non zero	2
BF_3	sp^2	Trigonal planar	Zero	0
SiF_4	sp^3	Tetrahedral	Zero	0
SF_4	sp^3d	Distorted square pyramidal (see-saw)	Non zero	1
XeF_4	sp^3d^2	Square planar	Zero	2

11.(A) $Ph-CHO + Ph-\underset{\underset{O}{\parallel}}{C}-CH_3 \xrightarrow[\Delta]{OH^-} Ph-CH=CH-\underset{\underset{O}{\parallel}}{C}-Ph$

(Benzalacetophenone)

$R-MgX \xrightarrow[2.\ H^+]{1.\ CO_2\,(\text{dry ice})} R-COOH$

(Step-up Reaction)

12.(C) Asparagine is a neutral amino acid
Cysteine comprises of -SH group
Glutamic acid and aspartic acid are non-essential amino acids

13.(C) Basic strength of amines depends on hydrogen bonding and electronic inductive effect.

$NH(Me)_2 > NH_2Me > NMe_3 > NH_3 > NH_2-NH_2$

14.(A) Theory-based

15.(D) $K_3[Fe(CN)_6]$ Paramagnetic (n = 1)

$K_2[CoCl_4]$ Paramagnetic (n = 3)

$Na_2[Ni(CN)_4]$ Diamagnetic; dsp^2 (Square planar geometry)

16.(C) Nitrite ions will form brown ring complex with dil. H_2SO_4, conc. H_2SO_4 and even dil. CH_3COOH so (II), (III) and (IV) are correct. Oxidation state of iron changes from +2 to +1 during the Brown ring test, therefore (I) is incorrect.

17.(B) $$\underset{20\,\text{m mol}}{2\,NH_4OH} + \underset{10\,\text{m mol}}{H_2SO_4} \longrightarrow (NH_4)_2SO_4 + 2H_2O(\ell)$$

mmol of $N = 20$

wt. of $N = \frac{20}{1000} \times 14 = \frac{28}{100} = \frac{7}{25}$ gm

% of $N = \frac{7}{25} \times \frac{1000}{14} = 20\%$

18.(A) Ease of reaction: 2º allylic > 1º allylic > 3º > 2º > 1º > vinylic

19.(B) Reactivity order: [Acid Halide > Acid anhydride > Ester > Amide]

20.(D) Given A is rarest, monoatomic, non-radioactive p-block element and form AX_2Y_3 type of molecule.

$\therefore$ It is concluded that it is Xe

It is given the electronegativity of A is less than X and Y

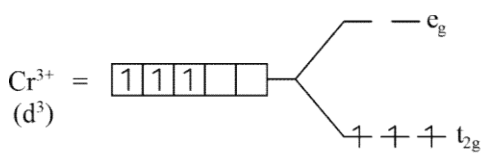
It is given that the electronegativity of X & Y is highest and second highest respectively among all elements.

$\therefore$ X & Y are F & O respectively. $\therefore$ Compound is XeO_3F_2 with trigonal bipyramidal geometry.

SECTION – 2

1.(0) The standard electrode potential $\left(M^{3+}/M^{2+}\right)$ is lowest for Cr among Cr, Mn, Fe and Co.

For the $\left[CrF_6\right]^{3-}$



Number of electrons in e_g orbitals = 0

2.(6) $E^\circ_{cell} = 0.86 - 0.7 = 0.16V$

$E_{cell} = E^\circ_{cell} - \frac{0.06}{2}\log\left[h^+\right]^2$

$0.46 = 0.16 - 0.06\,\log\left[H^+\right]$

$\log\left[H^+\right] = -5$ or $\left[H^+\right] = 10^{-5}M$

Now,

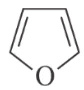
$$NH_4^+(aq) + H_2O(\ell) \rightleftharpoons NH_4OH(aq) + H^+(aq)\ ;\ K_h = \frac{K_w}{K_b}$$

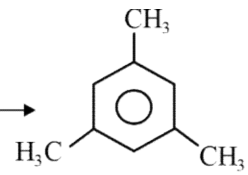
$t = 0$	$0.01M$	PURE	–	–
$t = t_{eq}$	$(0.01-x)M$ $\simeq 0.01M$	LIQUID	xM	xM

$\frac{x^2}{0.01} = \frac{K_w}{K_b}\ ;\ \frac{(10^{-5})^2}{0.01} = \frac{10^{-14}}{K_b}\ ;\ K_b = 10^{-6} \Rightarrow pK_b = 6$

3.(308) Molar mass of mesitylene $= 120\,\text{g mol}^{-1}$

0.2 mol weighs $= 0.2 \times 120 = 24\,\text{g}$

Furan $\longrightarrow$ 

Mesitylene $\longrightarrow$ 

Molar mass of furan $= 68\,\text{g mol}^{-1}$

0.1 mol weighs $= 0.1 \times 68 = 6.8\,\text{g}$

Value of $(x + y) = 24 + 6.8 = 30.8\ \text{g} = 308 \times 10^{-1}\text{g}$

4.(31)

$$\begin{array}{cccc} X_{2(g)} & + \ 3Y_{2(g)} & \rightleftharpoons 2XY_{3(g)} & \\ a & 0.1 & - & \\ a-x & 0.1-3x & 2x & \\ a-0.02 & 0.1-0.06 & 0.04 & \quad 2x = 0.04,\ x = 0.02\,\text{mol} \end{array}$$

So, $\left(n_{Y_2}\right)_{eq} = 0.04$ mol ; $\left(n_{XY_3}\right)_{eq} = 0.04$ mol

Now,

$PV = nRT$

$4 \times 2 = n(0.08)(250)$

$n = 0.4$ mol

$a - 0.02 + 0.1 - 0.06 + 0.04 = 0.4$

$a = 0.34\,\text{mol}$

So, $\left(n_{X_2}\right)_{eq} = 0.34 - 0.02 = 0.32$ mol

$$\therefore \qquad K_c = \frac{[XY_3]^2}{[X_2][Y_2]^3} = \frac{\left(\frac{0.04}{2}\right)^2}{\left(\frac{0.32}{2}\right)\left(\frac{0.04}{2}\right)^3} = 312.5 = 31.25 \times 10 \simeq 31 \times 10$$

5.(26) $t_{1/2}$ is independent of $[A]_0$, so the order of reaction is first order.

$$t_{1/2} = \frac{0.69}{k} = \text{intercept}$$

$$\frac{0.69}{k} = 25;\ k = \frac{0.69}{25}\,\text{min}^{-1}$$

$$\text{Now} \quad k = \frac{2.3}{t}\log\left(\frac{[A]_0}{[A]_t}\right); \quad \frac{0.69}{25} = \frac{2.3}{50}\log\left(\frac{[A]_0}{[A]_t}\right)$$

$$0.6 = \log\left(\frac{[A]_0}{[A]_t}\right);\ \log 4 = \log\left(\frac{[A]_0}{[A]_t}\right) = \log\left(\frac{3.5}{[A]_t}\right)$$

$$[A]_t = \frac{3.5}{4} = 0.875\ \text{mol L}^{-1}$$

Concentration of A remaining after 50 min. = $0.875\ \text{mol L}^{-1}$

Concentration of A consumed upto 50 min. $= 3.5 - 0.875 = 2.625\,\text{mol L}^{-1}$

$$= 26.25 \times 10^{-1}\,\text{mol L}^{-1} \simeq 26 \times 10^{-1}\,\text{mol L}^{-1}$$

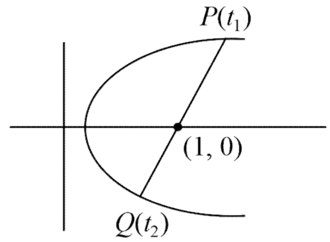
MATHEMATICS

SECTION – 1

1.(A) $P:\left(t_1^2,\,2t_1\right), \quad a=1$

$Q:\left(t_2^2,\,2t_2\right)$

$C:\dfrac{t_1^2+t_2^2}{2},\dfrac{2(t_1+t_2)}{2}$



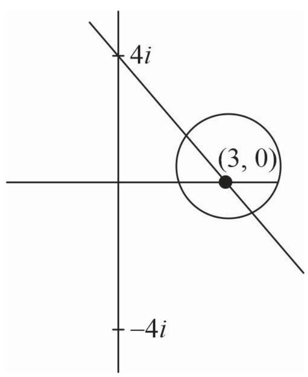
$y=\dfrac{-4}{\sqrt{5}} \Rightarrow t_1+t_2=\dfrac{-4}{\sqrt{5}}$

$PQ=\sqrt{\left(t_1^2-t_2^2\right)^2+\left(2t_1-2t_2\right)^2}$

$PQ=\sqrt{\left(t_1-t_2\right)^2(t_1+t_2)^2+4\left(t_1-t_2\right)^2}$

$PQ=\sqrt{\left((t_1+t_2)^2-4t_1t_2\right)\left[\dfrac{16}{5}+4\right]}=\sqrt{\left(\dfrac{16}{5}+4\right)\left(\dfrac{36}{5}\right)}=\sqrt{\dfrac{36}{5}\times\dfrac{36}{5}}=\dfrac{36}{5}$

2.(A) $(x-iy)(4+3i)+(x+iy)(4-3i)\le 24$



$8x+6y\le 24$

$4x+3y\le 12$

Minimum distance of (0, 4) from circle $=\sqrt{3^2+4^2}-1=4$ will lie along line joining (0, 4) & (3, 0) ;

$\dfrac{x}{3}+\dfrac{y}{4}=4$

Equation of circle $(x-3)^2+y^2=1 \Rightarrow y=\pm\dfrac{4}{5}$

Minimum distance $y=\dfrac{4}{5}$

$\therefore x=\dfrac{12}{5}$

$10\left(\dfrac{4}{5}+\dfrac{12}{5}\right)=32$

3.(C) $(2\sin\theta+1)(2\sin\theta-\sqrt{3})=0$

$\sin\theta=\frac{-1}{2},\frac{\sqrt{3}}{2}$ Number of solutions = 8

4.(C) $P(a\cos\theta, b\sin\theta)$: Tangent $\Rightarrow \frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

$PS\cdot PS'=a^2(1-e^2\cos\theta)$

$$d^2=\frac{1}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}=\frac{a^2(1-e^2)}{1-e^2\cos^2\theta}$$

$PS\times PS'\times d^2=a^2\,a^2(1-e^2)=a^2\,b^2=36$

5.(D) $\Delta BCD=\sqrt{8^2+9^2+10^2}=\sqrt{64+81+100}=\sqrt{245}$

6.(C) $\sum x_i-10\alpha=2$

$\sum x_i=10\alpha+2$

$\bar{x}=\frac{10\alpha+2}{10}=\frac{6}{5}\Rightarrow 10\alpha+2=12\Rightarrow \alpha=1$ …(1)

$\sum x_i^2+10\beta^2-2\beta\sum x_i=40$

$\sum x_i^2+10\beta^2-2\beta\times 12=40$

$\Rightarrow \sum x_i^2=40+24\beta-10\beta^2$ …(2)

Variance $=\frac{\sum x_i^2}{10}-(\bar{x})^2=\frac{84}{25}$

$\sum x_i^2=10\left(\frac{36}{25}+\frac{84}{25}\right)=\frac{10\times 120}{25}=48$

Putting in (2) we get

$5\beta^2-2\beta-10\beta+4=0$

$\beta(5\beta-2)-2(5\beta-2)=0$

$\beta=2,\frac{2}{5}$ (reject as β is positive integer)

Distance from (1, 6) is $\frac{|5+18-7|}{\sqrt{34}}\approx 2.75$

7.(B) $\lim_{x\to 0}\frac{\sin(ax)-x-\ln\cos x}{bx^2}=\frac{1}{2}$

$\frac{a\cos(ax)-1+\sec x\times\sin x}{2bx}\Rightarrow a-1=0$ [L-H rule]

$a=1$

$\frac{-a^2\sin(ax)+\sec^2 x}{2b}=\frac{1}{2}\Rightarrow b=1=2$

8.(B) $f'(x) = 6x^2 - 30ax + 36a^2$

$f'(x) = 6\left(x^2 - 5ax + 6a^2\right)$

$f'(x) = 6(x-2a)(x-3a)$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 2a & & 3a & \end{array}$$

$p = 2a$

$q = 3a$

$p^2 = 4q \Rightarrow 4a^2 = 4 \times 3a \Rightarrow a = 3$

$f(x) = 2x^3 - 45x^2 + 324x - 48$

$f(2) = 2 \times 8 - 45 \times 4 + 324 \times 2 - 48 = 436$

9.(A) Case I: $6, 6, 6, 6, 6, 2 \Rightarrow \dfrac{6!}{5!} = 6$

Case II : $6, 6, 6, 6, 4, 4 \Rightarrow \dfrac{6!}{4!2!} = 15$

Total ways $= 6 + 15 = 21$

10.(D) $2a + 2b = 4\left(2 + \sqrt{7}\right) \Rightarrow a + b = 4 + 2\sqrt{7}$

$e^2 = 1 + \dfrac{b^2}{a^2} = \dfrac{11}{4} \Rightarrow b^2 = \dfrac{7}{4}a^2$

$b = \dfrac{\sqrt{7}}{2}a$

$a\left(\dfrac{1}{1} + \dfrac{\sqrt{7}}{2}\right) = 4 + 2\sqrt{7}$

$a\left(2 + \sqrt{7}\right) = 8 + 4\sqrt{7}$

$a\left(2 + \sqrt{7}\right) = 4\left(2 + \sqrt{7}\right)$

$a = 4 \Rightarrow a^2 = 16$

$b = \dfrac{\sqrt{7}}{2} \times \sqrt{16} = 2\sqrt{7}$; $b^2 = 28$; $a^2 + b^2 = 44$

11.(A) $\vec{r} = \left(\vec{b}\right)\lambda + t\left(\vec{c}\right) \qquad \lambda, t \in R$

$= \left(\hat{i} + 2\hat{j} - \hat{k}\right)\lambda + t(\hat{i} + \hat{j} + 2\hat{k})$

$= (\lambda + t)\hat{i} + (2\lambda + t)\hat{j} - (\lambda + 2t)\hat{k}$

$\dfrac{\vec{r} \cdot \vec{a}}{\left|\vec{a}\right|} = \sqrt{\dfrac{2}{3}}$

$\left|(\lambda + t)\right| = 2 \Rightarrow \lambda + t = \pm 2$

$\Rightarrow \quad \vec{r} = (2, 2 + \lambda,\ \lambda - 4)$ or $(-2,\ \lambda - 2,\ \lambda + 4)$

Option (1) works with $\lambda = 1$.

12.(B) Note: If the given conditions are satisfied by (a, b), they are also satisfied by ($-a$, $-b$).

Enumerating cases for $a \geq 0$

a	b
0	0
1	1, 2
2	3
3	4, 5
4	6
5	7

Final answer = $1+7\times 2$ {To cover the elements where $a<0$ }

13.(B) $x=2\lambda-1$; $y=3\lambda-2$; $z=2\lambda+1$ (coordinates of Q)

$PQ=\sqrt{26}$

$(2\lambda-5)^2+(3\lambda-4)^2+(2\lambda-6)^2=26$

$4\lambda^2+25-20\lambda+9\lambda^2+16-24\lambda+4\lambda^2+36-24\lambda=26$

$17\lambda^2-68\lambda+51=0$

$17(\lambda^2-4\lambda+3)\Rightarrow \lambda=1,\ 3$

$Q:(1,1,3)$; $R:(5,7,7)$

Area $=\dfrac{1}{2}|PQ\times PR|=\dfrac{1}{2}\left|\left(3\hat{i}+\hat{j}+4\hat{k}\right)\times(\hat{i}+5\hat{j})\right|=\dfrac{1}{2}\left|\hat{k}-4\hat{j}-15\hat{k}+20\hat{i}\right|$

$=\dfrac{1}{2}\left|20\hat{i}-4\hat{j}-14\hat{k}\right|=\left|10\hat{i}-2\hat{j}-7\hat{k}\right|=\sqrt{153}$

14.(C) $f(x+y)=2^x f(y)+4^y f(x)$ …(i)

$f(y+x)=2^y f(x)+4^x f(y)$ …(ii)

$\dfrac{f(x)}{4^x-2^x}=\dfrac{f(y)}{4^y-2^y}=\lambda \Rightarrow f(x)=\left(4^x-2^x\right)\lambda$

$f(1)=2 \Rightarrow \lambda=1$; $f(x)=4^x-2^x$

$f'(x)=4^x\ln 4-2^x\ln 2$

$f'(2)=16\ln 4-4\ln 2=28\ln 2$; K = 28

15.(A) $R_3\to R_3+R_1$

$$|A|=(\alpha+\beta+\gamma)\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ 1 & 1 & 1\end{vmatrix}\ ;\ |A|=(\alpha+\beta+\gamma)(\alpha-\beta)\ (\beta-\gamma)\ (\gamma-\alpha)$$

$|adj\ A|=|A|^{n-1}$; $|adj\ adj\ (A)|=|A|^{(n-1)^2}$

$|adj\ (adj\,(adj\,(adj\,(A))))|=|A|^{(n-1)^4}=|A|^{2^4}=|A|^{16}$

$$\Rightarrow \frac{|A|^{16}}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=(\alpha+\beta+\gamma)^{16}$$

$6^{16}=2^{16}\times 3^{16}$; $m+n=32$

16.(A) $T_{r+1} = {}^{10}C_r (ax)^{10-r}\left(\frac{-1}{x}\right)^r$; $10-2r=0 \Rightarrow r=5$

$${}^{10}C_5\, a^{10-5}(-1)^5 = -8064 ; \quad \frac{10!}{5!5!}a^5 = 8064 \Rightarrow a^5 = 32 ; \quad a = 2 ; \; a^3 = 8$$

17.(B) $\alpha^2 = \alpha - 2$; $\beta^2 = \beta - 2$

$$\alpha^{n-2}\cdot\alpha^2 = \alpha^{n-2}(\alpha-2) \Rightarrow \alpha^n = \alpha^{n-1} - 2\alpha^{n-2}$$

$$\beta^{n-2}\cdot\beta^2 = \beta^{n-2}(\beta-2) \Rightarrow \beta^n = \beta^{n-1} - 2\beta^{n-2}$$

$$S_n = 2024(\alpha^{n-1} - 2\alpha^{n-2}) + 2025\,(\beta^{n-1} - 2\beta^{n-2})$$

$$S_n = 2024\cdot\alpha^{n-1} - 2025\,\beta^{n-1} - 2(2024\,\alpha^{n-2} + 2025\,\beta^{n-2})$$

$$S_n = S_{n-1} - 2S_{n-2}$$

$$S_{10} = S_9 - 2S_8$$

18.(C) Note: $A(i)\times A(j) = \begin{bmatrix}1 & 2i-1\\ 0 & 1\end{bmatrix}\times\begin{bmatrix}1 & 2j-1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & (2i-1+2j-1)\\ 0 & 1\end{bmatrix}$

Hence: $\prod_{k=1}^{50} A(k) = \begin{bmatrix}1 & \sum_{k=1}^{50} 2k-1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 2500\\ 0 & 1\end{bmatrix}$

19.(B) Given $27\,pqr \ge (p+q+r)^2$

But using $AM \ge GM \Rightarrow \quad \frac{p+q+r}{3} \ge (pqr)^{1/3} \quad \Rightarrow \quad p = q = r \qquad$ (using (i) and(ii))

Also, $3p + 4q + 5r = 12 \Rightarrow p = q = r = 1$

20.(A) $\Delta = 0 \Rightarrow \begin{vmatrix}1 & 2 & -1\\ 3 & 9 & \beta\\ 5 & 2 & 1\end{vmatrix} = 0$; $(9-2\beta) - 2(3-5\beta) - 1(6-45) = 0$

$$8\beta + 42 = 0 \Rightarrow \beta = \frac{-21}{4}$$

$$\Delta_1 = 0 \Rightarrow \begin{vmatrix}\alpha & 2 & -1\\ -2 & 9 & \frac{-21}{4}\\ 1 & 2 & 1\end{vmatrix} = 0 \Rightarrow \alpha = \frac{-1}{3}$$

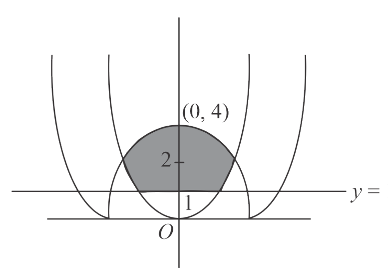
$$3\alpha - 4\beta = -1 + 21 = 20$$

SECTION – 2

1.(7) Area $= 2\times\left(\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right)$

$$= \frac{4}{3}\left(4\sqrt{2}-1\right) = \frac{16\sqrt{2}}{3} - \frac{4}{3}; \quad \alpha = 3 ; \beta = \frac{4}{3}$$

$$\alpha + 3\beta = 3 + 3\times\frac{4}{3} = 7$$



2.(24) $\sin 45° = \dfrac{r_2}{12+r_2}$

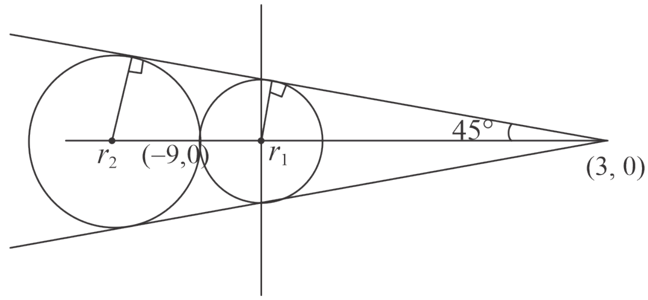
$12 + r_2 = \sqrt{2} r_2 \Rightarrow r_2 = \dfrac{12}{\sqrt{2}-1}$

$\sin 45° = \dfrac{r_1}{12-r_1}$

$r_1 = \dfrac{12}{\sqrt{2}+1}$

$|r_1 - r_2| = \left|\dfrac{12}{1+\sqrt{2}} - \dfrac{12}{\sqrt{2}-1}\right|$

$= \left|\dfrac{12(\sqrt{2}-1) - 12\left(\sqrt{2}+1\right)}{2-1}\right| = 24$



3.(45) $0 < \dfrac{\log x}{3} < 1 \Rightarrow 0 < \log x < 3 \quad \Rightarrow 1 < x < e^3 \Rightarrow \left[\dfrac{\log x}{3}\right] = 0$

$\Rightarrow 1 < \dfrac{\log x}{3} < 2 \Rightarrow e^3 < x < e^6; \quad \left[\dfrac{\log x}{3}\right] = 1$

$\int_1^{e^3} 0 \cdot dx + \int_{e^3}^{e^6} 1 \cdot dx = e^6 - e^3; \quad a^2 + b^2 = 36 + 9 = 45$

4.(6) $6f(x) = 3(f(x) + xf'(x)) - 3x^2$

$f(x) = y\ ;\ f'(x) = \dfrac{dy}{dx}; \quad y = x\dfrac{dy}{dx} - x^2; \quad \dfrac{dy}{dx} - \dfrac{y}{x} = x$

$P = -\dfrac{1}{x}, Q = x; \quad I \cdot F = e^{-\ln x} = \dfrac{1}{x}; \quad \dfrac{1}{x} \times y = \int x \cdot \dfrac{1}{x} + c$

$\dfrac{y}{x} = x + c; \quad y = x^2 + cx \Rightarrow f(1) = 2; \quad \Rightarrow c = 1$

$f(x) = x^2 + x$

$f(2) = 4 + 2 = 6$

5.(199) $3^a, 3^b, 3^c$

$(3^b)^2 = 3^a \cdot 3^c \Rightarrow 2b = a + c; \quad a, b, c \rightarrow A \cdot P$

$a + c \rightarrow \text{even} + \text{even} \text{ or } \text{odd} + \text{odd}$

$50_{C_2} \quad + \quad 51_{C_2}$

$= 1275 + 1225 = 2500$

Probability $= \dfrac{2500}{101_{C_3}} = \dfrac{2500}{101!} \times 3! \times 98! = \dfrac{2500 \times 6 \times 9}{101 \times 100 \times 9 \times 33} = \dfrac{50}{3333} = \dfrac{50 + 3333}{17} = 199$