Joint Entrance Exam | Mains-2019

Paper Code -

9th April 2019 | Evening

PHYSICS , CHMISTRY & MATHEMATICS

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360**.

5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics** and **Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.

6. Candidate will be awarded marks as stated above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks of the total marks allotted to the questions (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. For writing particulars/marking responses on *Side-1* and *Side-2* of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Page **20-23**) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***

12. The CODE for this Booklet is **B.** Make sure that the CODE printed on Side-2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. **Do not fold or make any stray mark on the Answer Sheet.**

Joint Entrance Exam/IITJEE-2019

PART-A	PHYSICS

1. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be:

(1) $\frac{M\omega_0}{M+6m}$ **(2)** $\frac{M\omega_0}{M+3m}$ **(3)** $\frac{M\omega_0}{M+2m}$ **(4)** $\frac{M\omega_0}{M+m}$

2. The area of a square is $5.29\ cm^2$. The area of 7 such squares taking into account the significant figures is:

(1) $37.030\ cm^2$ **(2)** $37\ cm^2$ **(3)** $37.03\ cm^2$ **(4)** $37.0\ cm^2$

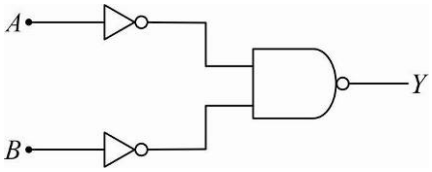
3. In a conductor, if the number of conduction electrons per unit volume is $8.5\times10^{28}m^{-3}$ and mean free time is 25 *fs* (femto second), it's approximate resistivity is: $(m_e = 9.1\times10^{-31}kg)$

(1) $10^{-7}\Omega m$ **(2)** $10^{-6}\Omega m$ **(3)** $10^{-8}\Omega m$ **(4)** $10^{-5}\Omega m$

4. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$, where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:

(1) $a+\frac{b^2}{c}$ **(2)** $a+\frac{b^2}{3c}$ **(3)** $a+\frac{b^2}{4c}$ **(4)** $a+\frac{b^2}{2c}$

5. The logic gate equivalent to the given logic circuit is:



(1) NAND **(2)** AND **(3)** NOR **(4)** OR

6. Diameter of the objective lens of a telescope is 250 *cm*. For light of wavelength 600 *nm* coming from a distant object, the limit of resolution of the telescope is close to:

(1) $1.5\times10^{-7}rad$ **(2)** $2.0\times10^{-7}rad$ **(3)** $4.5\times10^{-7}rad$ **(4)** $3.0\times10^{-7}rad$

7. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of $20\ ms^{-1}$ with respect to the ground. If an observer in car A detects a frequency 2000 *Hz* of the sound coming from car B, what is the natural frequency of the sound source in car B?
(Speed of sound in air $= 340\ ms^{-1}$)

(1) 2060 *Hz* **(2)** 2300 *Hz* **(3)** 2250 *Hz* **(4)** 2150 *Hz*

8. Four point charges $-q$, $+q$, $+q$ and $-q$ are placed on y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the x-axis at $x = D$, with $D >> d$, will behave as:

(1) $E \propto \frac{1}{D^4}$ **(2)** $E \propto \frac{1}{D}$ **(3)** $E \propto \frac{1}{D^2}$ **(4)** $E \propto \frac{1}{D^3}$

9. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} *N-m/rad*. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 *mA*. The value of *B* (in Tesla) is approximately:

(1) 10^{-1} **(2)** 10^{-4} **(3)** 10^{-3} **(4)** 10^{-2}

10. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r)=\frac{K}{r^2}$. Identify the correct relation between the radius *R* of the particle's orbit and its period *T*:

(1) *T*/*R* is constant **(2)** *TR* is a constant

(3) T^2/R^3 is a constant **(4)** T/R^2 is a constant

11. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on $1m^2$ surface area will be close to $(c=3\times10^8\ \text{m/s})$:

(1) $10\times10^{-8}N$ **(2)** $20\times10^{-8}N$ **(3)** $35\times10^{-8}N$ **(4)** $15\times10^{-8}N$

12. The position vector of a particle changes with time according to the relation $\vec{r}(t)=15t^2\hat{i}+(4-20t^2)\hat{j}$. What is the magnitude of the acceleration at *t* = 1?

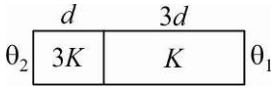
(1) 100 **(2)** 50 **(3)** 25 **(4)** 40

13. A string 2.0 *m* long and fixed at its ends is driven by a 240 *Hz* vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is:

(1) 320 *m*/*s*, 80 *Hz* **(2)** 180 *m*/*s*, 120 *Hz*

(3) 320 *m*/*s*, 120 *Hz* **(4)** 180 *m*/*s*, 80 *Hz*

14. Two materials having coefficients of thermal conductivity '3 *K*' and '*K*' and thickness '*d*' and '3 *d*', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, $(\theta_2>\theta_1)$. The temperature at the interface is:



(1) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$ **(2)** $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$

(3) $\frac{\theta_2+\theta_1}{2}$ **(4)** $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$

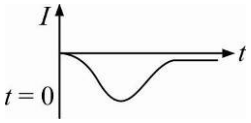
15. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be:

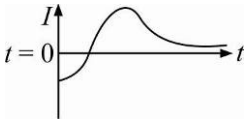
(1) $\frac{5}{3}\Omega$ **(2)** $\frac{5}{2}\Omega$ **(3)** $\frac{7}{2}\Omega$ **(4)** $\frac{12}{5}\Omega$

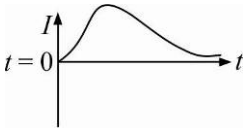
16. A particle of mass '*m*' is moving with speed '2*v*' and collides with a mass '2*m*' moving with speed '*v*' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass '*m*', which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be:

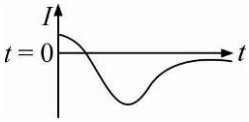
(1) $2\sqrt{2}\,v$ **(2)** $v/(2\sqrt{2})$ **(3)** $\sqrt{2}\,v$ **(4)** $v/\sqrt{2}$

17. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}(k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by:

(1) 

(2) 

(3) 

(4) 

18. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 $J/kg\ K$, specific heat of water = 4184 $J/kg\ K$)

(1) $10^{-1}K$ **(2)** $10^{-4}K$ **(3)** $10^{-3}K$ **(4)** $10^{-5}K$

19. Movement of inertia of a body about is given axis is 1.5 $kg\ m^2$. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of:

(1) 5 s **(2)** 2.5 s **(3)** 2s **(4)** 3 s

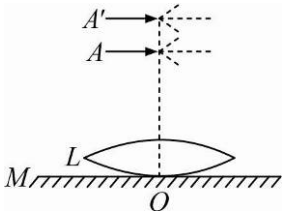
20. A He^+ ion is in its first excited state. Its ionization energy is:

(1) 6.04 eV **(2)** 48.36 eV **(3)** 54.40 eV **(4)** 13.60 eV

21. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distance x_1 and $x_2 (x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is:

(1) 4 : 3 **(2)** 5 : 3 **(3)** 3 : 1 **(4)** 2 : 1

22. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that $OA' = 27\ cm$, to get its inverted real image at A' itself. The value of μ_l will be:



(1) $\sqrt{3}$ **(2)** $\sqrt{2}$ **(3)** $\frac{4}{3}$ **(4)** $\frac{3}{2}$

23. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelength 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is:

(1) $\lambda_x - \lambda_y$ **(2)** $\lambda_x + \lambda_y$ **(3)** $\frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$ **(4)** $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$

24. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A?

(1) 0.02 ohm **(2)** 0.002 ohm **(3)** 0.02 ohm **(4)** 0.5 ohm

25. The physical sizes of the transmitter and receiver antenna in a communication system are:

(1) Independent of both carrier and modulation frequency
(2) Inversely proportional to carrier frequency
(3) Inversely proportional to modulation frequency
(4) Proportional to carrier frequency

26. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by:

(1) $\frac{2v^2}{5g}$ **(2)** $\frac{v^2}{g}$ **(3)** $\frac{2v^2}{7g}$ **(4)** $\frac{v^2}{2g}$

27. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of $10^{-3} Wb$ passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is:

(1) $3.67\times10^{-4}\ Wb$ **(2)** $6.67\times10^{-3}\ Wb$
(3) $3.67\times10^{-3}\ Wb$ **(4)** $6.67\times10^{-4}\ Wb$

28. A wooden block floating in a bucket of water has 4/5 of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is:

(1) 0.7 **(2)** 0.8 **(3)** 0.5 **(4)** 0.6

29. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is:

(1) $\frac{(n+1)V}{(K+n)}$ **(2)** $\frac{V}{K+n}$ **(3)** V **(4)** $\frac{nV}{K+n}$

30. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of $J\ \text{mol}^{-1}K^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then:

(1) A is rigid but B has a vibrational mode
(2) Both A and B have a vibrational mode each
(3) A has on vibrational mode and B has two
(4) A has a vibrational mode but B has none

PART-B	CHEMISTRY

1. The amorphous form of silica is:

(1) tridymite **(2)** kieselguhr **(3)** cristobalite **(4)** quartz

2. Among the following species, the diamagnetic molecule is:

(1) CO **(2)** O_2 **(3)** NO **(4)** B_2

3. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?

(1) 0.15 **(2)** 0.05 **(3)** 0.20 **(4)** 0.10

4. In the following reaction, carbonyl compound +MeOH $\xrightarrow{HCl}$ acetal. Rate of the reaction is the highest for:

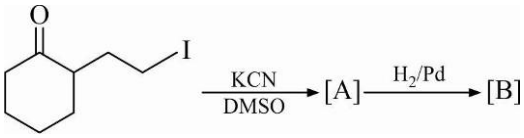
(1) Acetone as substrate and methanol in excess

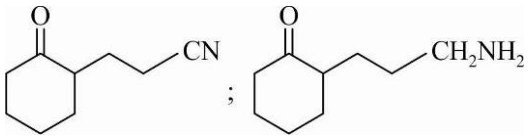
(2) Propanal as substrate and methanol in excess

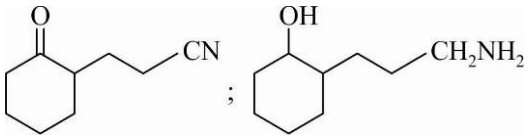
(3) Propanal as substrate and methanol in stoichiometric amount

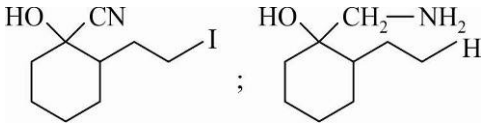
(4) Acetone as substrate and methanol in stoichiometric amount

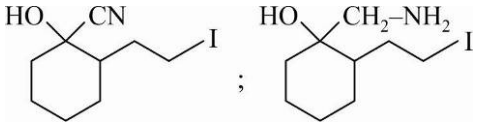
5. The major products A and B for the following reactions are, respectively:

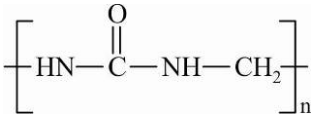


(1) 

(2) 

(3) 

(4) 

6. Which of the following compounds is a constituent of the polymer 

?

(1) Ammonia **(2)** N-Methyl urea **(3)** Formaldehyde **(4)** Methylamine

7. Noradrenaline is a/an

(1) Neurotransmitter **(2)** Antihistamine

(3) Antidepressant **(4)** Antacid

8. **Assertion :** For the extraction of iron, haematite ore is used.

Reason: Haematite is a carbonate ore of iron

(1) Only the assertion is correct

(2) Both the assertion and reason are correct and the reason is the correct explanation for the assertion

(3) Only the reason is correct

(4) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion

9. The maximum number of possible oxidation states of actinoides are shown by:

(1) neptunium (Np) and plutonium (Pu)

(2) actinium (Ac) and thorium (Th)

(3) nobelium (No) and lawrencium (Lr)

(4) berkelium (Bk) and californium (Cf)

10. What would be the molality of 20% (mass/mass) aqueous solution of KI?

(molar mass of KI = 166 g mol^{-1})

(1) 1.48 **(2)** 1.51 **(3)** 1.08 **(4)** 1.35

11. The correct statements among I to III are:

(I) Valence bond theory cannot explain the color exhibited by transition metal complexes

(II) Valence bond theory can predict quantitatively the magnetic properties of transition metal complexes

(III) Valence bond theory cannot distinguish ligands as weak and strong field ones

(1) (I), (II) and (III) **(2)** (I) and (II) only

(3) (I) and (III) only **(4)** (II) and (III) only

12. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behavior.

Their equation of state is given as $P = \frac{RT}{V - b}$ at T. Here, b is the van der Waals constant. Which gas will exhibit steepest increase in the plot of Z (compression factor) vs P ?

(1) Xe **(2)** Ne **(3)** Kr **(4)** Ar

13. The correct statements among I to III regarding group 13 element oxides are,

(I) Boron trioxide is acidic.

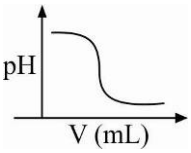
(II) Oxides of aluminium and gallium are amphoteric

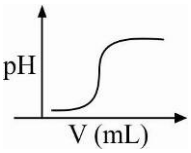
(III) Oxides of indium and thallium are basic

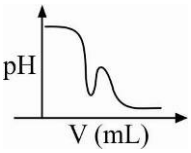
(1) (I), (II) and (III) **(2)** (I) and (II) only

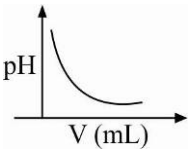
(3) (II) and (III) only **(4)** (I) and (III) only

14. In an acid-base titration, 0.1 M HCl solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titration mixture in this experiment?

(A) 

(B) 

(C) 

(D) 

(1) (C) **(2)** (A) **(3)** (D) **(4)** (B)

15. HF has highest boiling point among hydrogen halides, because it has:

(1) strongest hydrogen bonding **(2)** lowest ionic character

(3) strongest van der Waal's interactions **(4)** lowest dissociation enthalpy

16. During compression of a spring the work done is 10 kJ and 2 kJ escaped to the surroundings as heat. The change in internal energy, ΔU (in kJ) is

(1) 8 **(2)** – 12 **(3)** 12 **(4)** – 8

17. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect? (The Bohr radius is represented by a_0) :

(1) The electron can be found at a distance $2a_0$ from the nucleus

(2) The magnitude of the potential energy is double that of its kinetic energy on an average

(3) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

(4) The probability density of finding the electron is maximum at the nucleus

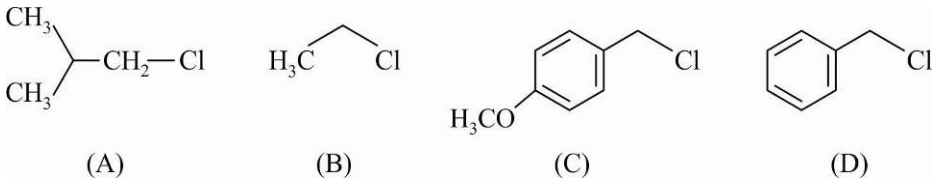
18. Hinsberg's reagent is:

(1) $C_6H_5SO_2Cl$ **(2)** $SOCl_2$ **(3)** C_6H_5COCl **(4)** $(COCl)_2$

19. The peptide that gives positive ceric ammonium nitrate and carbylamines tests is:

(1) Lys - Asp **(2)** Ser – Lys **(3)** Asp - Gln **(4)** Gln - Asp

20. Increasing order of reactivity of the following compounds for S_N1 substitution is:



(A) (B) (C) (D)

(1) (B) < (C) < (D) < (A) **(2)** (B) < (C) < (A) < (D)

(3) (A) < (B) < (D) < (C) **(4)** (B) < (A) < (D) < (C)

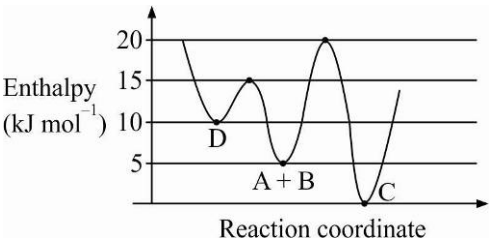
21. Molal depression constant for a solvent is 4.0 K kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is: (Assume complete dissociation of the electrolyte)

(1) 0.18 K **(2)** 0.24 K **(3)** 0.36 K **(4)** 0.12 K

22. Consider the given plot of enthalpy of the following reaction between *A* and *B*.

$A + B \rightarrow C + D$.

Identify the incorrect statement.



(1) C is the thermodynamically stable product

(2) Activation enthalpy to form C is 5 kJ mol^{-1} less than that to form D

(3) Formation of A and B from C has highest enthalpy of activation

(4) D is kinetically stable product

23. The one that is not a carbonate ore is:

(1) bauxide **(2)** malachite **(3)** siderite **(4)** calamine

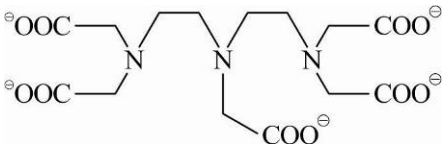
24. The layer of atmosphere between 10 km to 50 km above the sea level is called as:

(1) mesosphere **(2)** thermosphere **(3)** stratosphere **(4)** troposphere

25. 10 mL of 1 mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as a cube, what is its edge length?

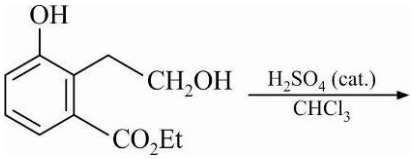
(1) 1.0 pm **(2)** 0.1 nm **(3)** 2.0 nm **(4)** 2.0 pm

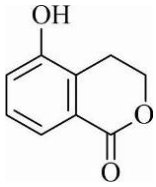
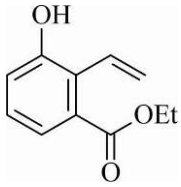
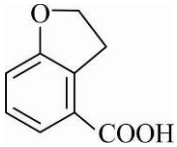
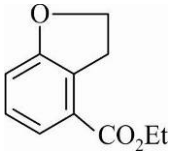
26. The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are:



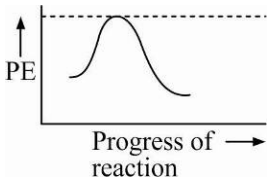
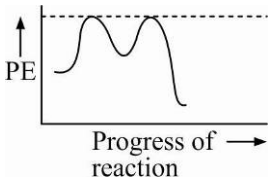
(1) 6 and 8 **(2)** 8 and 8 **(3)** 6 and 6 **(4)** 8 and 6

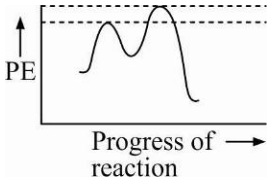
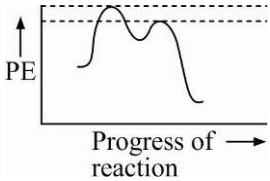
27. The major product of the following reaction is:



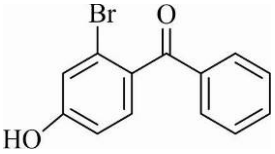
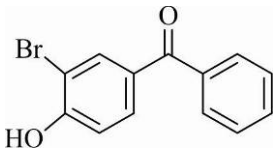
(1)  **(2)**  **(3)**  **(4)** 

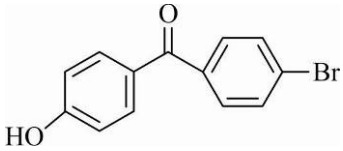
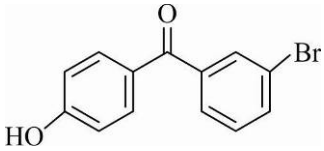
28. Which of the following potential energy (PE) diagrams represents the S_N1 reaction?

(1)  **(2)** 

(3)  **(4)** 

29. p-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives:

(1)  **(2)** 

(3)  **(4)** 

30. The structures of beryllium chloride in the solid state and vapour phase, respectively, are:

(1) dimeric and chain **(2)** dimeric and dimeric

(3) chain and chain **(4)** chain and dimeric

PART-C	MATHEMATICS

1. Let $z \in C$ be such that $|z| < 1$. If $\omega = \dfrac{5+3z}{5(1-z)}$, then:

(1) $4\,\mathrm{Im}(\omega) > 5$ **(2)** $5\,\mathrm{Im}(\omega) < 1$ **(3)** $5\,\mathrm{Re}(\omega) > 4$ **(4)** $5\,\mathrm{Re}(\omega) > 1$

2. The common tangent to the circles $x^2+y^2=4$ and $x^2+y^2+6x+8y-24=0$ also passes through the point:

(1) $(4,-2)$ **(2)** $(-4,6)$ **(3)** $(-6,4)$ **(4)** $(6,-2)$

3. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx$ is:

(1) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$ **(2)** $\frac{\pi}{4}-\log_e 2$ **(3)** $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ **(4)** $\frac{\pi}{2}-\log_e 2$

4. Two newspapers *A* and *B* are published in a city. It is known that 25% of the city population reads *A* and 20% reads *B* while 8% reads both *A* and *B*. Further, 30% of those who read *A* but not *B* look into advertisements and 40% of those who read *B* but not *A* also look into advertisements, while 50% of those who read both *A* and *B* look into advertisements. Then the percentage of the population who look into advertisements is:

(1) 12.8 **(2)** 13.5 **(3)** 13.9 **(4)** 13

5. The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)$ is:

(1) $(1,2)\cup(2,\infty)$ **(2)** $(-2,-1)\cup(-1,0)\cup(2,\infty)$

(3) $(-1,0)\cup(1,2)\cup(3,\infty)$ **(4)** $(-1,0)\cup(1,2)\cup(2,\infty)$

6. If the system of equations $2x+3y-z=0$, $x+ky-2z=0$ and $2x-y+z=0$ has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:

(1) $\frac{1}{2}$ **(2)** $-\frac{1}{4}$ **(3)** $\frac{3}{4}$ **(4)** -4

7. If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta\in(0,\pi)$ with $\hat{k}$, then a value of θ is:

(1) $\frac{5\pi}{12}$ **(2)** $\frac{5\pi}{6}$ **(3)** $\frac{\pi}{4}$ **(4)** $\frac{2\pi}{3}$

8. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consist of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is:

(1) 190 **(2)** 225 **(3)** 262 **(4)** 157

9. The vertices *B* and *C* of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that $BC=5$ units. Then the area (in sq. units) of this triangle, given that the point $A(1,-1,2)$, is

(1) $2\sqrt{34}$ **(2)** $\sqrt{34}$ **(3)** 6 **(4)** $5\sqrt{17}$

10. If the function $f(x)=\begin{cases}a|\pi-x|+1, & x\le 5\\ b|x-\pi|+3, & x>5\end{cases}$ is continuous at $x=5$, then the value of $a-b$ is:

(1) $\frac{2}{\pi-5}$ **(2)** $\frac{2}{\pi+5}$ **(3)** $\frac{2}{5-\pi}$ **(4)** $\frac{-2}{\pi+5}$

11. Two poles standing on a horizontal ground are of heights 5 *m* and 10 *m* respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in *m*) between the poles, is:

(1) $10(\sqrt{3}-1)$ **(2)** $5(\sqrt{3}+1)$ **(3)** $\frac{5}{2}(2+\sqrt{3})$ **(4)** $5(2+\sqrt{3})$

12. If the tangent to the parabola $y^2 = x$ at a point $(\alpha, \beta), (\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to:

(1) $2\sqrt{2}+1$ **(2)** $\sqrt{2}+1$ **(3)** $2\sqrt{2}-1$ **(4)** $\sqrt{2}-1$

13. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$. Upto 11^{th} term is:

(1) 945 **(2)** 946 **(3)** 916 **(4)** 915

14. If m is chosen in the quadratic equation $(m^2+1)x^2 - 3x + (m^2+1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

(1) $8\sqrt{5}$ **(2)** $4\sqrt{3}$ **(3)** $8\sqrt{3}$ **(4)** $10\sqrt{5}$

15. If $f : R \to R$ is a differentiable function and $f(2) = 6$, then $\lim\limits_{x\to 2} \int\limits_{6}^{f(x)} \frac{2t\ dt}{(x-2)}$ is:

(1) $24f'(2)$ **(2)** $12f'(2)$ **(3)** $2f'(2)$ **(4)** 0

16. If $\int e^{\sec x}(\sec x \tan x\ f(x) + (\sec x \tan x + \sec^2 x))dx = e^{\sec x} f(x) + C$, then a possible choice of $f(x)$ is:

(1) $\sec x + x\tan x - \frac{1}{2}$ **(2)** $x\sec x + \tan x + \frac{1}{2}$

(3) $\sec x - \tan x - \frac{1}{2}$ **(4)** $\sec x + \tan x + \frac{1}{2}$

17. Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to:

(1) $\frac{17}{\sqrt{5}}$ **(2)** $63\sqrt{5}$ **(3)** $\frac{11}{\sqrt{5}}$ **(4)** $205\sqrt{5}$

18. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are (– 8, 5) and (6, 5), then the area of the rectangle (in sq. units) is:

(1) 84 **(2)** 56 **(3)** 72 **(4)** 98

19. The area (in sq. units) of the region $A = \{(x, y) : y^2/2 \le x \le y + 4\}$ is:

(1) 18 **(2)** $\frac{53}{3}$ **(3)** 16 **(4)** 30

20. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m, is:

(1) $\frac{2}{\pi}$ **(2)** $\frac{1}{15\pi}$ **(3)** $\frac{1}{5\pi}$ **(4)** $\frac{1}{10\pi}$

21. If the two lines $x + (a - 1)y = 1$ and $2x + a^2 y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is:

(1) $\frac{2}{\sqrt{5}}$ **(2)** $\frac{\sqrt{2}}{5}$ **(3)** $\sqrt{\frac{2}{5}}$ **(4)** $\frac{2}{5}$

22. If $\cos x \frac{dy}{dx} - y\sin x = 6x, \left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{\pi^2}{4\sqrt{3}}$ **(2)** $-\frac{\pi^2}{2}$ **(3)** $-\frac{\pi^2}{2\sqrt{3}}$ **(4)** $\frac{\pi^2}{2\sqrt{3}}$

23. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is:

(1) – 25 **(2)** – 36 **(3)** – 35 **(4)** 25

24. If some three consecutive coefficients in the binomial expansion of $(x+1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is:

(1) 232 **(2)** 625 **(3)** 964 **(4)** 227

25. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is:

(1) $4\pi(2-\sqrt{2})$ **(2)** $8\pi(2-\sqrt{2})$ **(3)** $8\pi(3-2\sqrt{2})$ **(4)** $4\pi(3+\sqrt{2})$

26. If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively:

(1) T, F, F **(2)** F, T, T **(3)** F, F, F **(4)** T, T, F

27. The value of sin10° sin30° sin50° sin70° is:

(1) $\frac{1}{32}$ **(2)** $\frac{1}{18}$ **(3)** $\frac{1}{36}$ **(4)** $\frac{1}{16}$

28. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to:

(1) $\frac{8}{3}$ **(2)** $\frac{7}{3}$ **(3)** $\frac{9}{4}$ **(4)** $\frac{7}{2}$

29. The total number of matrices $A=\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}, (x, y \in R, x \neq y)$ for which $A^T A = 3I_3$ is:

(1) 4 **(2)** 3 **(3)** 6 **(4)** 2

30. If $f(x) = [x] - \left[\frac{x}{4}\right], x \in R,$ where $[x]$ denotes the greatest integer function, then:

(1) $\lim\limits_{x \to 4+} f(x)$ exists but $\lim\limits_{x \to 4-} f(x)$ does not exit

(2) Both $\lim\limits_{x \to 4-} f(x)$ and $\lim\limits_{x \to 4+} f(x)$ exist but are not equal

(3) $\lim\limits_{x \to 4-} f(x)$ exists but $\lim\limits_{x \to 4+} f(x)$ does not exist

(4) f is continuous at $x = 4$