Joint Entrance Exam | Mains-2019

Paper Code -

12th April 2019 | Evening

PHYSICS , CHMISTRY & MATHEMATICS

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360**.

5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics** and **Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.

6. Candidate will be awarded marks as stated above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks of the total marks allotted to the questions (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. For writing particulars/marking responses on *Side-1* and *Side-2* of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Page **20-23**) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***

12. The CODE for this Booklet is **B.** Make sure that the CODE printed on Side-2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. **Do not fold or make any stray mark on the Answer Sheet.**

Joint Entrance Exam/IITJEE-2019

PART-A	PHYSICS

1. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$).

(1) $\frac{b^2\tau}{\sqrt{2}}$ **(2)** $\frac{b^2\tau}{4}$ **(3)** $\frac{b^2\tau}{2}$ **(4)** $b^2\tau$

2. Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charge A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance, a from the centre. If A and B do not experience any force, then:

(1) $a = 2^{-1/4}R$ **(2)** $a = \frac{R}{\sqrt{3}}$ **(3)** $a = 8^{-1/4}R$ **(4)** $a = \frac{3R}{2^{1/4}}$

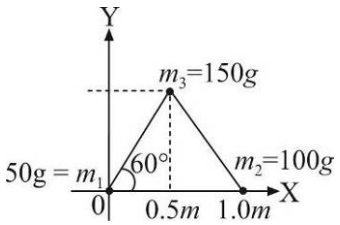
3. Two source of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed um/s and he hears 10 beat/s. Then, u is equal to:

(1) $5.5\ m/s$ **(2)** $15.0\ m/s$ **(3)** $10.0\ m/s$ **(4)** $2.5\ m/s$

4. A cannot engine has an efficiency of 1/6. When the temperature of the sink is reduced by $62°C$, its efficiency is doubled. The temperature of the source and the sink are respectively.

(1) $99°C, 37°C$ **(2)** $37°C, 99°C$ **(3)** $62°C, 124°C$ **(4)** $124°C, 62°C$

5. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be:



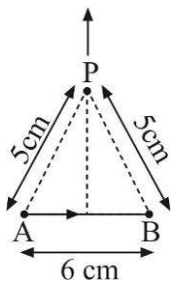
(1) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ **(2)** $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$

(3) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$ **(4)** $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

6. The number density of molecules of a gas depends on their distance r from the origin as $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to:

(1) $n_0\alpha^{-3/4}$ **(2)** $\sqrt{n_0}\ \alpha^{1/2}$ **(3)** $n_0\alpha^{1/4}$ **(4)** $n_0\alpha^{-3}$

7. Find the magnetic field at point P due to a straight-line segment AB of length 6 cm carrying a current of 5A. (see figure) $(\mu_0 = 4\pi \times 10^{-7}\ N - A^{-2})$



(1) $3.0 \times 10^{-5}T$

(2) $2.5 \times 10^{-5}T$

(3) $2.0 \times 10^{-5}T$

(4) $1.5 \times 10^{-5}T$

8. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be:

(1) 1 : 8 **(2)** 3 : 8 **(3)** 8 : 1 **(4)** 9 : 8

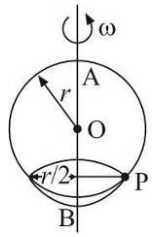
9. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of electricity equals T. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to:

(1) $3F/(\pi r^2 YT)$ **(2)** $6F/(\pi r^2 YT)$ **(3)** $F/(3\pi r^2 YT)$ **(4)** $9F/(\pi r^2 YT)$

10. A system of three polarizers P_1, P_2, P_3 is set p such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at $60°$ to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio $((I_0/I)$ equals (nearly):

(1) 10.67 **(2)** 1.80 **(3)** 16.00 **(4)** 5.33

11. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in the figure, the bead is at rest with respect to the circular ring at potion P as shown. Then the value of ω^2 is equal to:



(1) $\dfrac{\sqrt{3}g}{2r}$ **(2)** $2g/r$ **(3)** $2g/\left(r\sqrt{3}\right)$ **(4)** $\left(g\sqrt{3}\right)/r$

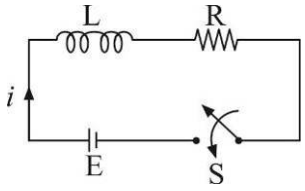
12. A small speaker delivers $2W$ of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Given reference intensity of sound as $10^{-12} W/m^2$]

(1) 30 cm **(2)** 20 cm **(3)** 40 cm **(4)** 10 cm

13. A diatomic gas with rigid molecules does $10J$ of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

(1) $30\,J$ **(2)** $40\,J$ **(3)** $25\,J$ **(4)** $35\,J$

14. Consider the LR circuit shown in the figure. If the switch S is closed at $t = 0$ then the amount of charge that passes through the battery between $t = 0$ and $t = \dfrac{L}{R}$ is:



(1) $\dfrac{EL}{2.7R^2}$ **(2)** $\dfrac{2.7EL}{R^2}$

(3) $\dfrac{EL}{7.3R^2}$ **(4)** $\dfrac{7.3EL}{R^2}$

15. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into

(i) An ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and

(ii) Into a voltmeter of range 0 to $V (V = GI_0)$ by connecting a series resistance R_V to it. Then,

(1) $R_A R_V = G^2$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$

(2) $R_A R_V = G^2$ and $\dfrac{R_A}{R_V} = \dfrac{I_g}{\left(I_0 - I_g\right)}$ $R_A R_V = G^2\left(\dfrac{I_g}{I_0 - I_g}\right)$

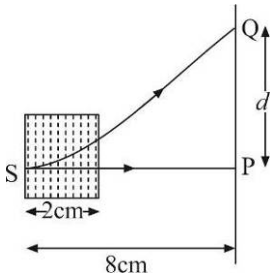
(3) $\dfrac{R_A}{R_V} = \left(\dfrac{I_0 - I_g}{I_g}\right)^2$ $R_A R_V = G^2\left(\dfrac{I_0 - I_g}{I_g}\right)$

(4) $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{\left(I_0 - I_g\right)}\right)^2$

16. The ratio of the weight of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}th$ of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density)

(1) $\frac{R}{4}$ **(2)** $\frac{R}{9}$ **(3)** $\frac{R}{3}$ **(4)** $\frac{R}{2}$

17. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3} T)\hat{k}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q(on the screen) is: (electron's charge $= 1.6 \times 10^{-19} C$, mass of electron $= 9.1 \times 10^{-31} kg$)



(1) 11.65 cm **(2)** 2.25 cm **(3)** 1.22 cm **(4)** 12.87 cm

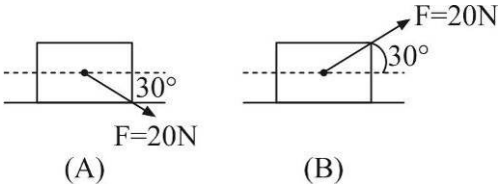
18. A tuning fork of frequency 480 Hz is used in an experiment of measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30\ cm$ and $l_2 = 70\ cm$. Then, v is equal to:

(1) 379 ms^{-1} **(2)** 332 ms^{-1} **(3)** 338 ms^{-1} **(4)** 384 ms^{-1}

19. A spring whose unstretched length is l has a force constant k. The spring is cut into two piece of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1 / k_2 of the corresponding force constants, k_1 and k_2 will be:

(1) $\frac{1}{n^2}$ **(2)** n^2 **(3)** $\frac{1}{n}$ **(4)** n

20. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20\ N$, making an angle of $30°$ with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be: $(g = 10 ms^{-2})$



(1) 0 ms^{-2} **(2)** 3.2 ms^{-2} **(3)** 0.4 ms^{-1} **(4)** 0.8 ms^{-1}

21. A plane electromagnetic wave having a frequency $v = 23.9\ GHz$ propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

(1) $\vec{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}$ **(2)** $\vec{B} = 2 \times 10^7 \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t)\hat{i}$

(3) $\vec{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t)\hat{j}$ **(4)** $\vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\hat{i}$

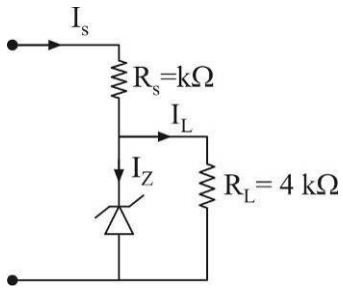
22. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65Å). The de-Broglie wavelength of the electron is:

(1) 12.9Å **(2)** 6.6Å **(3)** 9.7Å **(4)** 3.5Å

23. One kg of water, at $20°C$, heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is closed to: [Specific heat of water = 4200 J/(kg °C), Latent heat of water = 2260 kJ/kg]

(1) 3 minutes **(2)** 16 minutes **(3)** 22 minutes **(4)** 10 minutes

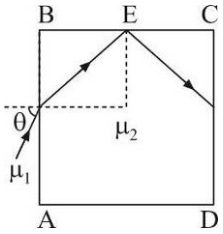
24. Figure shows a *DC* voltage regulator circuit, with a Zener diode of breakdown voltage = $6V$. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(1) 7.5 mA **(2)** 2.5 mA

(3) 1.5 mA **(4)** 3.5 mA

25. A transparent cube of side *d*, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AV at an angle θ (shown in the figure). Total internal refraction takes place at point *E* on the face *BC*. Then θ must satisfy:



(1) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ **(2)** $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

(3) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ **(4)** $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

26. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity v_2, when falling through the same fluid, the ratio (v_1 / v_2) equals:

(1) 1/27 **(2)** 27 **(3)** 9 **(4)** 1/9

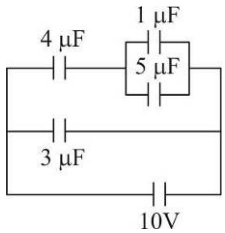
27. In an amplitude modulator circuit, the carrier wave is given by, $C(t) = 4\sin(20000\,\pi t)$ while modulating signal is given by, $m(t) = 2\sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are:

(1) 0.3 and 9 kHz **(2)** 0.5 and 10 kHz **(3)** 0.4 and 10 kHz **(4)** 0.5 and 9 kHz

28. Two particles are projected from the same point with the same speed *u* such that they have the same range *R*, but different maximum heights, h_1 and h_2. Which of the following is correct?

(1) $R^2 = 4h_1h_2$ **(2)** $R^2 = 2h_1h_2$ **(3)** $R^2 = h_1h_2$ **(4)** $R^2 = 16\,h_1h_2$

29. In the given circuit, the charge on $4\mu F$ capacitor will be:



(1) $13.4\mu C$

(2) $9.6\,\mu C$

(3) $5.4\mu C$

(4) $24\mu C$

30. The electron in a hydron atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1 / λ_2 of the photons emitted in this process is:

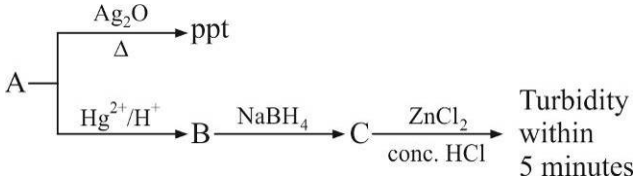
(1) 27/5 **(2)** 9/7 **(3)** 7/5 **(4)** 20/7

PART-B	CHEMISTRY

1. Among the following, the energy of 2s orbital is lowest in :

(1) K **(2)** Li **(3)** Na **(4)** H

2. Consider the following reactions :



'A' is :

(1) $CH_3-C\equiv C-CH_3$ **(2)** $CH_3-C\equiv CH$

(3) $CH_2=CH_2$ **(4)** $CH\equiv CH$

3. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z respectively, are :

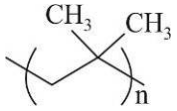
(1) $K_2MnO_4, KMnO_4$ and Cl_2 **(2)** $KMnO_4, K_2MnO_4$ and Cl_2

(3) $K_2MnO_4, KMnO_4$ and SO_2 **(4)** K_3MNO_4, K_2MnO_4 and Cl_2

4. The molar solubility of $Cd(OH)_2$ is $1.84\times10^{-5}M$ in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is :

(1) $\frac{2.49}{1.84}\times10^{-9}M$ **(2)** $2.49\times10^{-10}M$ **(3)** $6.23\times10^{-11}M$ **(4)** $1.84\times10^{-9}M$

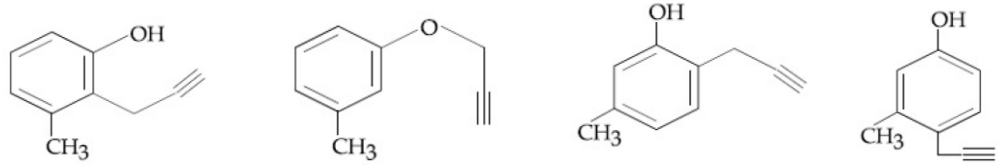
5. The correct name of the following polymer is :



(1) Polyisobutylene **(2)** Polytert-butylene

(3) Polyisobutane **(4)** Polyisoprene

6. What will be the major product when m-cresol is reacted with propargyl bromide $(HC\equiv C-CH_2Br)$ in presence of K_2CO_3 in acetone ?

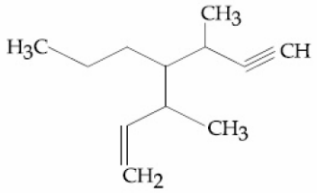


(1) **(2)** **(3)** **(4)**

7. The pair that has similar atomic radii is :

(1) Mo and W **(2)** Mn and Re **(3)** Ti and Hf **(4)** Sc and Ni

8. The IUPAC name for the following compound is :



(1) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene

(2) 3,5-dimethyl-4-propylhept-1-en-6-yne

(3) 3,5-dimethyl-4-propylhept-6-en-1-yne

(4) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

9. The compound used in the treatment of lead poisoning is .

(1) Cis-platin **(2)** D-penicillamine
(3) desferrioxime B **(4)** EDTA

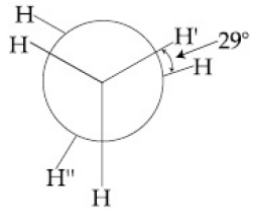
10. The primary pollutant that leads to photochemical smog is :
(1) nitrogen oxides **(2)** Sulphur dioxide
(3) ozone **(4)** acrolein

11. Among the following, the INCORRECT statement about colloids is :
(1) They can scatter light
(2) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration
(3) The range of diameters of colloidal particles is between 1 and 1000 nm
(4) They are larger than small molecules and have high molar mass

12. The INCORRECT match in the following is :
(1) $\Delta G^\circ > 0, K < 1$ **(2)** $\Delta G^\circ < 0, K < 1$ **(3)** $\Delta G^\circ = 0, K = 1$ **(4)** $\Delta G^\circ < 0, K > 1$

13. An 'Assertion' and a 'Reason' are given below. Choose the correct answer from the following options :
Assertion(A) : Vinyl halides do not undergo nucleophilic substitution easily
Reason (R) : Even though the intermediate carbocation is stabilized by loosely held $\pi-$ electrons, the cleavage is difficult because of strong bonding
(1) Both (A) and (R) are correct statements and (R) is the correct explanation of (A)
(2) Both (A) and (R) are wrong statements
(3) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A)
(4) (A) is correct statement but (R) is a wrong statement

14. In the following skew conformatioin of ethane, $H' - C - C - H''$ dihedral angle is :



(1) 120° **(2)** 149° **(3)** 58° **(4)** 151°

15. The C–C bond length is maximum in :
(1) C_{60} **(2)** graphite **(3)** C_{70} **(4)** diamond

16. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$. The initial concentration of N_2O_5 is 3.00 $mol\,L^{-1}$ and it is 2.75 $mol\,L^{-1}$ after 30 minutes. The rate of formation of NO_2 is :
(1) $2.083 \times 10^{-3}\, mol\,L^{-1}\, min^{-1}$ **(2)** $4.167 \times 10^{-3}\, mol\,L^{-1}\, min^{-1}$
(3) $1.667 \times 10^{-2}\, mol\,L^{-1}\, min^{-1}$ **(4)** $8.333 \times 10^{-3}\, mol\,L^{-1}\, min^{-1}$

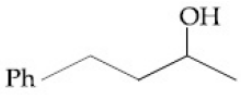
17. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 $g\,mol^{-1}$) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27°C. The osmotic pressure of the solution is :
(R = 0.08206 L atm $K^{-1}\, mol^{-1}$)
(1) 1.64 atm **(2)** 4.92 atm **(3)** 8.2 atm **(4)** 2.46 atm

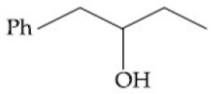
18. In which one of the following equilibria, $K_P \neq K_C$?

(1) $2NO(g) \quad N_2(g) + O_2(g)$ **(2)** $2C(s) + O_2(g) \quad 2CO(g)$

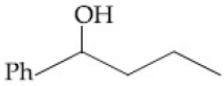
(3) $2HI(g) \quad H_2(g) + I_2(g)$ **(4)** $NO_2(g) + SO_2(g) \quad NO(g) + SO_3(g)$

19. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(PAc)_2 / H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is :

(1) 

(2) 

(3) 

(4) 

20. In comparison to boron, beryllium has :

(1) greater nuclear charge and lesser first ionization enthalpy

(2) greater nuclear charge and greater first ionization enthalpy

(3) lesser nuclear charge and greater first ionization enthalpy

(4) lesser nuclear charge and lesser first ionization enthalpy

21. The decreasing order of electrical conductivity of the following aqueous solution is :

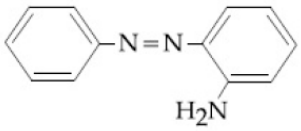
0.1 M forming acid (A),

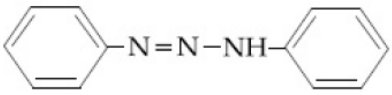
0.1 M Acetic acid (B),

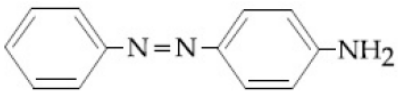
0.1 M Benzoic acid (C).

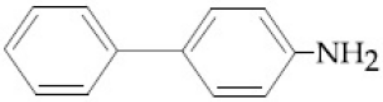
(1) C > B > A **(2)** C > A > B **(3)** A > B > C **(4)** A > C > B

22. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

(1) 

(2) 

(3) 

(4) 

23. Which one of the following is likely to give a precipitate with $AgNO_3$ solution ?

(1) CCl_4 **(2)** $(CH_3)_3CCl$ **(3)** $CHCl_3$ **(4)** $CH_2 = CH = Cl$

24. The correct statement is :

(1) the Hall-Heroult process is used for the production of aluminium and iron

(2) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate

(3) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2

(4) pig iron is obtained from cast iron

25. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively, are :

(1) $Mg(HCO_3)_2$ and $Mg(OH)_2$ **(2)** $Ca(HCO_3)_2$ and $Ca(OH)_2$

(3) $Ca(HCO_3)_2$ and $Ca(OH)_2$ **(4)** $Mg(HCO_3)_2$ and $MgCO_3$

26. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains :

(1) 20 g of carbon and 5 g of hydrogen **(2)** 22 g of carbon and 3 g of hydrogen
(3) 18 g of carbon and 7 g of hydrogen **(4)** 24 g of carbon and 1 g of hydrogen

27. The INCORRECT statement is :
(1) LiCl crystallises from aqueous solution as $LiCl \cdot 2H_2O$
(2) Lithium is the strongest reducing agent among the alkali metals
(3) Lithium is least reactive with water among the alkali metals
(4) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2

28. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are :
(en = ethane-1, 2-diamine)
(1) 5 and 3 **(2)** 6 and 6 **(3)** 5 and 6 **(4)** 3 and 3

29. Which of the given statements is INCORRECT about glycogen ?
(1) It is a straight chain polymer similar to amylose
(2) It is present in animal cells
(3) It is present in some yeast and fungi
(4) Only $\alpha-$ linkages are present in the molecule

30. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively :
(1) 4 : 2 : 1 **(2)** 8 : 1 : 6 **(3)** 4 : 2 : 3 **(4)** 1 : 2 : 4

PART-C	MATHEMATICS

1. A value of α such that $\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right)$ is:
(1) 2 **(2)** $-\frac{1}{2}$ **(3)** $\frac{1}{2}$ **(4)** –2

2. $\lim\limits_{x\to\infty} \frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is:
(1) 2 **(2)** 2 **(3)** 1 **(4)** 6

3. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to:
(1) 48 **(2)** 24 **(3)** $2\sqrt{6}$ **(4)** $4\sqrt{3}$

4. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?
(1) $\left(1, 2\sqrt{2}\right)$ **(2)** $\left(2, \sqrt{2}\right)$ **(3)** $\left(2, 2\sqrt{2}\right)$ **(4)** $\left(\sqrt{2}, 2\right)$

5. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r} = (\hat{i}+\hat{j}) + \lambda(i+2\hat{j}-\hat{k})$ and $\vec{r} = (i+\hat{j}) + \mu(-\hat{i}+\hat{j}-2\hat{k})$ is:
(1) $\sqrt{3}$ **(2)** $\frac{1}{3}$ **(3)** 3 **(4)** $\frac{1}{\sqrt{3}}$

6. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?
(1) If $(A - C \subseteq)B$, then $A \subseteq B$ **(2)** $(C \cup A) \cap (C \cup B) = C$
(3) If $(A - B) \subseteq C$, then $A \subseteq C$ **(4)** $B \cap C \neq \phi$

7. The general solution of the differential equation $(y^2 - x^3)dx - xydy = 0 (x \neq 0)$ is: (where c is a constant of integration)

(1) $y^2 - 2x^3 + cx^2 = 0$ **(2)** $y^2 + 2x^3 + cx^2 = 0$

(3) $y^2 - 2x^2 + cx^3 = 0$ **(4)** $y^2 + 2x^2 + cx^3 = 0$

8. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

(1) (2, 4, 1) **(2)** (1, 4, –1) **(3)** (2, –4, 1) **(4)** (1, –4, 1)

9. A value of $\theta \in \left(0, \frac{\pi}{3}\right)$, for which $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is:

(1) $\frac{\pi}{18}$ **(2)** $\frac{7\pi}{36}$ **(3)** $\frac{7\pi}{24}$ **(4)** $\frac{\pi}{9}$

10. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to:

(1) $\beta\gamma$ **(2)** 0 **(3)** $\alpha\beta$ **(4)** $\alpha\gamma$

11. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is:

(1) 120 **(2)** 150 **(3)** 280 **(4)** 200

12. A person throws two fair dice. He wins Rs.15 for throwing a doublet (same numbers on the two dice), winds Rs.12 when the throw results in the sum of 9, and loses Rs.6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is:

(1) $\frac{1}{2}$ gain **(2)** $\frac{1}{4}$ loss **(3)** 2 gain **(4)** $\frac{1}{2}$ loss

13. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to:

(1) [1, 4] **(2)** [2, 6] **(3)** [3, 7] **(4)** R

14. If $^{20}C_1 + (2^2)\,^{20}C_2 + (3^2)\,^{20}C_3 + ... + (20^2)\,^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to:

(1) (380, 19) **(2)** (380, 18) **(3)** (420, 19) **(4)** (420, 18)

15. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60º with the line $x + y = 0$. Then an equation of the line L is:

(1) $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$ **(2)** $x + \sqrt{3}y = 8$

(3) $\left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 8\sqrt{2}$ **(4)** $\sqrt{3}x + y = 8$

16. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point:

(1) (3, 5) **(2)** (3, 10) **(3)** (1, 5) **(4)** (2, 3)

17. Let $\alpha \in R$ and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set $S = \{\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$:

(1) contains exactly two positive numbers

(2) contains exactly two numbers only one of which is positive

(3) is empty

(4) is singleton

18. The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$ with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is:

(1) 2 **(2)** $\frac{2}{3}$ **(3)** 1 **(4)** $\frac{1}{2}$

19. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|, x \in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim\limits_{x \to \beta} \frac{(x-1)(x^2 - 5x + 6)}{x^2 - 6x + 8}$ is equal to:

(1) $-\frac{1}{2}$ **(2)** $-\frac{3}{2}$ **(3)** $\frac{1}{2}$ **(4)** $\frac{3}{2}$

20. Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$ $A(x)\cos 2\alpha + B(x)\sin 2\alpha + C,$ where C is a constant of integration, then the functions $A(x)$ and $B(x)$ are respectively:

(1) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$ **(2)** $x + \alpha$ and $\log_e |\sin(x + \alpha)|$

(3) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$ **(4)** $x + \alpha$ and $\log_e |\sin(x - \alpha)|$

21. A group of students comprises of 5 boys and *n* girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girls in each team, is 1750, then *n* is equal to:

(1) 24 **(2)** 27 **(3)** 28 **(4)** 25

22. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is:

(1) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ **(2)** $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ **(3)** $\frac{134}{25}\left(\frac{1}{5}\right)^{48}$ **(4)** $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

23. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to:

(1) $(\sim p) \Rightarrow q$ **(2)** $p \vee q$ **(3)** $p \wedge q$ **(4)** $q \Rightarrow \sim p$

24. The term independent of *x* in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right) \cdot \left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to:

(1) 36 **(2)** –72 **(3)** –36 **(4)** –108

25. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is:

(1) $\left(\frac{1}{3}, \frac{5}{3}\right)$ **(2)** $\left(\frac{1}{3}, 2\right)$ **(3)** $\left(1, \frac{7}{3}\right)$ **(4)** $\left(\frac{1}{3}, 1\right)$

26. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45º from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the lower from B be 30º, then the distance (in m) of the foot of the tower from the point A is:

(1) $15\left(5-\sqrt{3}\right)$ **(2)** $15\left(3-\sqrt{3}\right)$ **(3)** $15\left(1+\sqrt{3}\right)$ **(4)** $15\left(3-\sqrt{3}\right)$

27. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x+[-\cos\theta]y=0$ $[\cot\theta]x+y=0$ has a unique solution if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$

(1) and have infinitely many solutions if $\theta\in\left(\pi,\frac{7\pi}{6}\right)$

(2) have infinitely many solutions if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(3) has a unique solution if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(4) have infinitely many solutions if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ and has a unique solution if $\theta\in\left(\pi,\frac{7\pi}{6}\right)$

28. The equation of a common tangent to the curve, $y^2=16x$ and $xy=-4$ is:

(1) $x-y+4=0$ **(2)** $x-2y+16=0$ **(3)** $2x-y+2=0$ **(4)** $x+y+4=0$

29. The tangents to the curve $y=(x-2)^2-1$ at its points of intersection with the line $x-y=3$, intersect at the point:

(1) $\left(\frac{5}{2},1\right)$ **(2)** $\left(-\frac{5}{2},-1\right)$ **(3)** $\left(-\frac{5}{2},1\right)$ **(4)** $\left(\frac{5}{2},1\right)$

30. Let $z\in C$ with $\text{Im}(z)=10$ and it satisfies $\frac{2z-n}{2z+n}=2i-1$ for some natural number n. then:

(1) $n=20$ and $\text{Re}(z)=10$ **(2)** $n=40$ and $\text{Re}(z)=10$

(3) $n=40$ and $\text{Re}(z)=-10$ **(4)** $n=20$ and $\text{Re}(z)=-10$