MATHEMATICS | ICSE SAMPLE PAPER

1.1.(B) Here, selling price of fan = ₹1650

GST on fan = 12% of ₹1650 $= 1650 \times \frac{12}{100} = 198$

Thus, cost of a fan to the consumer inclusive of tax = ₹(1650 + 198) = ₹1848

1.2.(A) We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get, $a = p^2, b = p^2 - q^2$ and $c = -q^2$

Now, discriminant, $D = b^2 - 4ac$

$$= (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2) = (p^2 - q^2)^2 + 4p^2q^2 = p^4 + q^4 - 2p^2q^2 + 4p^2q^2$$

$$= p^4 + q^4 + 2p^2q^2 = (p^2 + q^2)^2 > 0$$

So, the given equation has real roots, which are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(p^2 - q^2) + \sqrt{(p^2 + q^2)^2}}{2p^2} = \frac{-p^2 + q^2 + p^2 + q^2}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - \sqrt{(p^2 + q^2)^2}}{2p^2}$

$$= \frac{-p^2 + q^2 - (p^2 + q^2)}{2p^2} = \frac{-p^2 + q^2 - p^2 - q^2}{2p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, the required roots are $\frac{q^2}{p^2}$ and -1.

1.3.(B) Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + bx + a$

Since $x + 2$ is a common factor of $f(x)$ and $g(x)$

Then $f(-2) = 0$

$\Rightarrow (-2)^2 + a(-2) + b = 0 \qquad \Rightarrow 4 - 2a + b = 0 \qquad \ldots(1)$

and $g(-2) = 0$

$\Rightarrow (-2)^2 + b(-2) + a = 0 \qquad \Rightarrow 4 - 2b + a = 0 \qquad \ldots(2)$

equation (1) equation (2) gives;

$(4 - 2a + b) - (4 - 2b + a) = 0$

$\Rightarrow 4 - 2a + b - 4 + 2b - a = 0 \Rightarrow 3b - 3a = 0 \Rightarrow b = a \Rightarrow \frac{a}{b} = 1 \Rightarrow a : b = 1 : 1$

1.4.(C) It has $m \times n$ entries

Since a matrix A is of order $m \times n$ containing m rows and n columns and hence contains mn elements.

1.5.(B) Let $S = 1 + (1+2) + (1+2+3) + (1+2+3+4) + ... + (1+2+3+...+20)$

= 1 + 3 + 6 + 10 +15 +21 +28 + 36 + 45 + 55 + 66 + 78 + 91 + 105 +120 +153 +171 + 190 + 210 $= 1540 [\because 1+2+3+....+n = \frac{n(n+1)}{2}]$

1.6.(A) (8, 5)

$\because$ Image of (–8, 5) under origin $\equiv (8, -5)$ and image of (8, –5) under X-axis $\equiv (8,\ 5)$

$\therefore h = 8 \, and \, k = 5$

1.7.(A) $3\sqrt{17}\,cm$

Given $AB = 8cm$ and $BC = 6cm$

$\therefore AC = \sqrt{8^2 + 6^2} = 10cm$

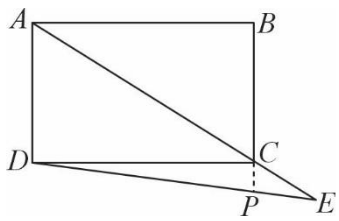
Also, given $AC : CE = 2:1$

Now, produce BC to meet DE at the point P as CP is parallel to AD,

$\Delta ECP \sim \Delta EAD$ ...(i)

$\Rightarrow \frac{CP}{AD} = \frac{CE}{AE} \Rightarrow \frac{CP}{6} = \frac{1}{3}$ ...(ii)

$\Rightarrow CP = 2cm$



Also, ΔCPD is right triangle.

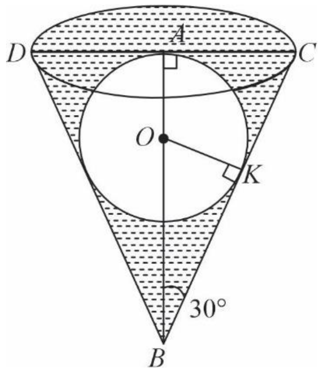
$\therefore DP = \sqrt{CD^2 + CP^2} = \sqrt{68} = 2\sqrt{17}\ cm$

But $DP = PE = 2:1$ [from Eq. (i)]

$\therefore PE = \sqrt{17}\ cm$

Thus, $DE = DP + PE = 2\sqrt{17} + \sqrt{17} = 3\sqrt{17}\ cm$

1.8.(a) $\frac{5\pi}{3}a^3$



Let radius of sphere be a, i.e. $OK = OA = a$

Then, the centre O of a sphere will be centroid of the ΔBCD

$\because OA = \frac{1}{3}AB \Rightarrow AB = 3(OA)$

In right angles ΔOKB,

$$\sin 30° = \frac{OK}{OB} = \frac{a}{OB} \Rightarrow \frac{1}{2} = \frac{a}{OB}$$

$\Rightarrow \quad OB = 2a$

Now, $AB = OA + OB = a + 2a = 3a$

Now, in right angled ΔBAC,

$$\frac{AC}{AB} = \tan 30° \Rightarrow \frac{AC}{AB} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad AC = \sqrt{3}a$ units

Now, volume of a cone $BCD = \frac{1}{3}\pi(AC)^2 \times AB = \frac{1}{3}\pi(a\sqrt{3})^2 \times 3a = 3\pi a^3$

$\therefore$ Volume of water remaining in the cone = Volume of the cone BCD – Volume of a sphere

$= 3\pi a^3 - \frac{4}{3}\pi a^3 = \frac{5\pi}{3} a^3$ *cu* units

1.9.(b) $\{x : x \in (2, 4) \cup (4, 6), x \in R\}$

Given inequation is $\left|\frac{2}{x-4}\right| > 1, x \neq 4$

$\Rightarrow \quad 1 < \frac{2}{x-4}$ or $\frac{2}{x-4} < -1$ [splitting the inequation]

Consider $\frac{2}{x-4} > 1$. If $x - 4 > 0$

Then, $2 > x - 4$ [multiplying by (x – 4) on both sides]

$\Rightarrow \quad x < 6$

So, $\quad 4 < x < 6$

If $\quad x - 4 < 0$. Then, $2 < x - 4$ [multiplying by (x – 4) both sides]

$\Rightarrow \quad x > 6$

But, $x < 4$ and $x > 6$ is not possible simultaneously.

Consider $\frac{2}{x-4} < -1$

If $x - 4 < 0$. Then, $2 > -(x-4)$ [multiplying by (x – 4) both sides]

$\Rightarrow \quad 2 > -x + 4 \Rightarrow -x < -2 \Rightarrow x > 2$

So, $\quad 2 < x < 4$.

If $x - 4 > 0$. Then, $2 < -(x-4)$ [multiply by (x – 4) on both sides]

$\Rightarrow \quad 2 < -x + 4 \Rightarrow -x > -2 \Rightarrow x < 2$

But, $x < 2$ and $x > 4$ is not possible simultaneously. The required solution set is $\{x : x \in (2, 4) \cup (4, 6), x \in R\}$.

1.10.(d) $\frac{3}{8}$

When three different coins are tossed simultaneously

Then $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore \quad n(S) = 8$

E : event of getting exactly one head $= \{HTT, THT, TTH\}$ $\therefore$ $n(E) = 3$

Thus $P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$

1.11.(b) $2X$

$$X^2 = X \cdot X = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+0 & -2-6 \\ 0+0 & 0+9 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$$

$$X^2 - 2X + 3I = \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix} - 2\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+3 & -8+4+0 \\ 0-0+0 & 9-6+3 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 6 \end{bmatrix} = 2\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = 2X$$

1.12.(c) $(0, 3-4\sqrt{3})$

Let the third vertex of an equilateral triangle be (x, y).

Then, vertices of triangles are $A(-4, 3), B(4, 3)$ and $C(x, y)$

We know that in equilateral triangle, the angle between two adjacent sides is 60° and all three sides are equal.

$\therefore \quad AB = BC = CA$

$\Rightarrow \quad AB^2 = BC^2 = CA^2 \qquad \ldots(i)$

Now taking first two terms, $AB^2 = BC^2$

$\Rightarrow \quad (4+4)^2 + (3-3)^2 = (x-4)^2 + (y-3)^2$ $\left[\because \text{distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\right]$

$\Rightarrow \quad 64 + 0 = x^2 + 16 - 8x + y^2 + 9 - 6y \left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$\Rightarrow \quad x^2 + y^2 - 8x - 6y = 39 \qquad \ldots(ii)$

Now, taking first and third terms, $AB^2 = CA^2$

$\Rightarrow \quad (4+4)^2 + (3-3)^2 = (-4-x)^2 + (3-y)^2$

$\Rightarrow \quad 64 + 0 = 16 + x^2 + 8x + 9 + y^2 - 6y$

$\Rightarrow \quad x^2 + y^2 + 8x - 6y = 39 \qquad \ldots(ii)$

On subtracting Eq. (ii) from Eq. (iii), we get

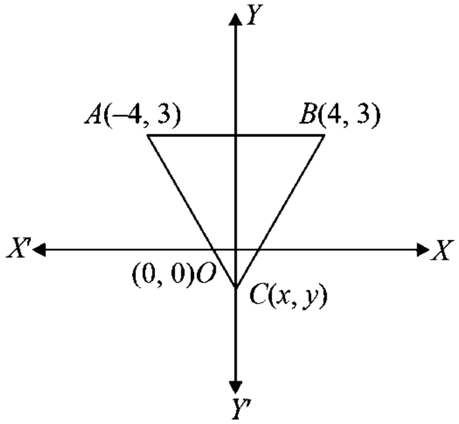
$$x^2+y^2+8x-6y=39$$
$$x^2+y^2-8x-6y=39$$
$$-\quad -\quad +\quad +\quad -$$
$$16x=0$$
$\Rightarrow \quad x=0$

Now, putting the value of x in Eq. (ii), we get

$$0+y^2-0-6y=39$$
$$\Rightarrow \quad y^2-6y-39=0$$
$$\therefore \quad y=\frac{6\pm\sqrt{(-6)^2-4(1)(-39)}}{2\times 1} \quad \text{[by quadratic formula, } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\text{]}$$
$$=\frac{6\pm\sqrt{36+156}}{2}=\frac{6\pm\sqrt{192}}{2}=\frac{6\pm 2\sqrt{48}}{2}=6\pm\sqrt{48}$$
$$=3\pm 4\sqrt{3}=3+4\sqrt{3} \text{ or } 3-4\sqrt{3}$$

So, the points of third vertex are $(0,3+4\sqrt{3})$ or $(0,3-4\sqrt{3})$.

But given that, the origin lies in the interior of the ΔABC and the x-coordinate of third vertex is zero. Then, y-coordinate of third vertex should be negative.



Hence, the required coordinates of third vertex are $C(0,3-4\sqrt{3})$ $[\because C\neq(0,3+4\sqrt{3})]$

1.13.(b) 18

We know that,

tangents drawn from external point to the circle are of equal length.

$\therefore \quad EK=EM=9\,cm \quad \ldots(1)$

Let the tangent DF meets the given circle at N

From external point D;

$DK=DN \quad \ldots(2)$

From external point F;

$FM=FN \quad \ldots(3)$

$\therefore$ perimeter of $EDF=ED+EF+DF=(EK-DK)+(EM-FM)+DN+FN$

$=9\,cm+9\,cm-DK-FM+DN+FN=18\,cm$ [using (2) and (3)]

1.14 **(D) 25.5**

Explanation : As x, y and $2x$ are in ascending order, therefore median is y.

$\therefore\ y = 27$

Also, mean $= \dfrac{x+y+2x}{3} = 33 \Rightarrow \dfrac{3x+27}{3} = 33 \Rightarrow x = 24$

$\therefore$ Mean of x and $y = \dfrac{x+y}{2} = \dfrac{24+27}{2} = \dfrac{51}{2} = 25.5$

1.15. **(A)**

Both A and R are true and R is the correct explanation of A.

Explanation: If a, b, c are in AP, then

$b - a = c - b$

$a + c = 2b$

For $\dfrac{1}{bc}, \dfrac{1}{ac}, \dfrac{1}{ab}$

$\dfrac{1}{ac} - \dfrac{1}{bc} = \dfrac{b-a}{abc}$

$\dfrac{1}{ab} - \dfrac{1}{ac} = \dfrac{c-b}{abc}$

From above two equations

$\dfrac{1}{bc}, \dfrac{1}{ac}, \dfrac{1}{ab}$ are also in AP.

And their common difference is divided by abc since numbers are also divided by abc.

Question 2

2.1 Deposit per month (P) = Rs 200

Rate (R) = 10% p.a.

Maturity value = Rs 6775

Let period = n months

$\therefore$ Principal for 1 month $= P \times \dfrac{n(n+1)}{2} = \dfrac{n(n+1)}{2} \times 200 = Rs\ 100n(n+1)$

Interest $= \dfrac{PRT}{100} = \dfrac{100n(n+1) \times 10 \times 1}{100 \times 12} = \dfrac{5n(n+1)}{6}$ ....(i)

But Interest = M.V. – Deposit

$= 6775 - 200 \times n$

$- 6775 - 200\,n$ ...(ii)

From (i) and (ii)

$$\frac{5n(n+1)}{6} = 6775 - 200n$$

$\Rightarrow 5n(n+1) = 40650 - 1200n \qquad \Rightarrow 5n^2 + 5n + 1200n - 40650 = 0$

$\Rightarrow 5n^2 + 1205n - 40650 = 0 \qquad \Rightarrow n^2 + 241n - 8130 = 0$

$$\Rightarrow n^2 - 30n + 271n - 8130 = 0 \begin{cases} \because 8130 = -30 \times 271 \\ 241 = -30 + 271 \end{cases}$$

$= n(n-30) + 271(n-30)$

$\Rightarrow (n-30)(n+271) = 0$

Either $n - 30 = 0$, then $n = 30$

Or $n + 271 = 0,\ n = -271$

which is not possible

$\therefore$ Period = n = 30 months $= 2\frac{1}{2}$ years

2.2 Given $y = \dfrac{10ab}{a+b}$

$$\Rightarrow \frac{y}{5a} = \frac{2b}{a+b}$$

Now apply componando and dividendo we get

$$\frac{y+5a}{y-5a} = \frac{2b+a+b}{2b-a-b};\ \frac{y+5a}{y-5a} = \frac{3b+a}{b-a}$$

Again $\dfrac{y}{5b} = \dfrac{2a}{a+b}$

Again, apply componendo and devidendo we get

$$\frac{y+5b}{y-5b} = \frac{2a+a+b}{2a-a-b};\ \frac{y+5b}{y-5b} = \frac{3a+b}{a-b}$$

$$\therefore \frac{y+5a}{y-5a} + \frac{y+5b}{y-5b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$= \frac{3b+a}{b-a} - \frac{3a+b}{b-a} = \frac{3b+a-3a-b}{b-a} = \frac{2b-2a}{b-a} = \frac{2(b-a)}{(b-a)} = 2$$

2.3 Prove that $\sec^2 A - \left(\dfrac{\sin^2 a - 2\sin^4 A}{2\cos^4 A - \cos^2 A}\right) = 1$

$$\text{LHS } = \sec^2 A - \left(\frac{\sin^2 A - 2\sin^4 A}{2\cos^4 A - \cos^2 A}\right) = \sec^2 A - \frac{\sin^2 A(1-2\sin^2 A)}{\cos^2 A(2\cos^2 A - 1)}$$

$$= \sec^2 A - \tan^2 A\frac{(1-2(1-\cos^2 A))}{(2\cos^2 A-1)} = \sec^2 A - \tan^2 A\frac{(1-2+2\cos^2 A)}{(2\cos^2 A-1)}$$

$$= \sec^2 A - \tan^2 A\frac{(2\cos^2 A-1)}{(2\cos^2 A-1)} = \sec^2 A - \tan^2 A$$

Question 3

3.1 Given, radius of cylinder $(r) = 7$ cm

Height (h) = 14 cm

radius of one sphere (R) = 3.5 cm

Let the number of spheres formed by n.

$\therefore$ Volume of n spheres formed = Volume of cylinder.

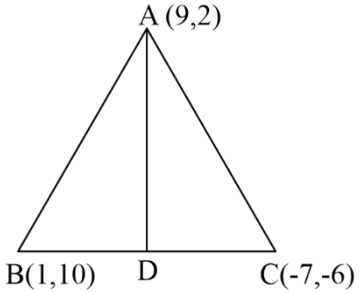
$$n \times \frac{4}{3}\pi R^3 = \pi r^2 h$$

$$n = \frac{4}{3} \times \pi \times (3.5)^3 = \pi \times 7 \times 7 \times 14$$

$$n = \frac{7 \times 7 \times 14 \times 3}{4 \times 3.5 \times 3.5 \times 3.5} = 12$$

Hence, number of spheres formed = 12.

3.2.



Median drawn through vertex 'A' bisect BC. i.e it intersect BC at their mid point.

$$\therefore \text{ mid-point of } BC = \left(\frac{1+(-7)}{2}, \frac{10+(-6)}{2}\right) = \left(\frac{-6}{2}, \frac{4}{2}\right) = (-3, 2)$$

Equation of median $(AD) \Rightarrow y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y-2=\dfrac{2-2}{-3-9}(x-9)$ $\Rightarrow y-2=\dfrac{0}{-12}(x-9)$ $\Rightarrow y-2=0$

Hence, equation of median $AD = y-2=0$

Now, Altitude from A is $\perp BC$

$\therefore$ Slope of $BC=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-6-10}{-7-1}=\dfrac{-16}{-8}=2$

$\therefore$ Slope of altitude $=\dfrac{-1}{2}$ $\{\because m_1m_2=-1\}$

$\therefore$ Equation of altitude through 'A' and having slope $=\dfrac{-1}{2}$ is given by $A(9, 2)$

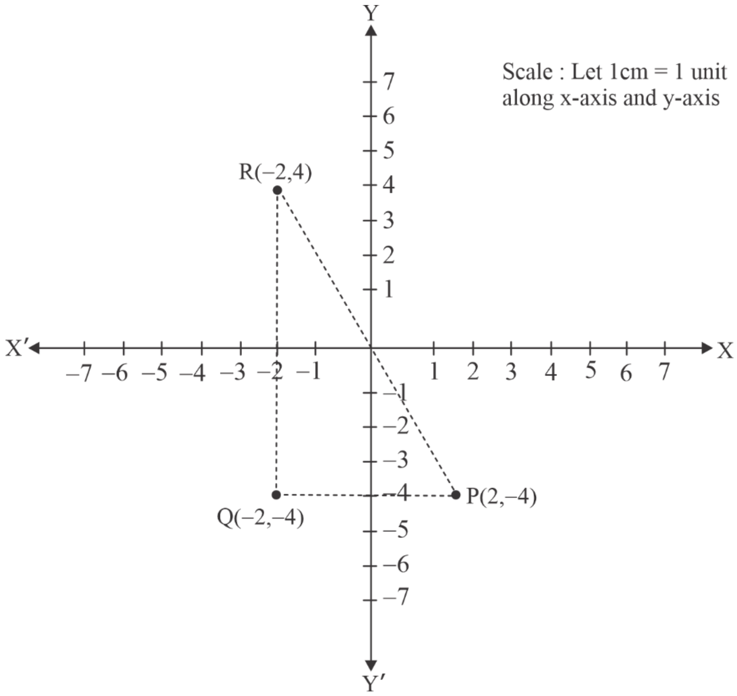
$\Rightarrow y-y_1=m(x-x_1)$ $\Rightarrow y-2=\dfrac{-1}{2}(x-9)$

$\Rightarrow y-2=-\dfrac{1}{2}x+\dfrac{9}{2}$ $\Rightarrow 2y-4=-x+9$

$2y+x-13=0$

Hence, equation of altitude through A is $2y-x-13=0$

3.3.



i. The coordinates of Q are (–2, –4)

ii. The coordinates of R are (–2, 4)

iii. PQR is right angle triangle

iv. Area of $\Delta PQR=\dfrac{1}{2}\times \text{Base}\times\text{height}=\dfrac{1}{2}\times 4\times 8$

$=2\times 8=16\ sq.\ unit$

Question 4

4.1. For dealer Vinod,

Selling price = Rs. 1000 (Given)

Since in case of inter-state, we get

IGST = Rs. $\frac{12}{100} \times 1000 = \text{Rs}.120$

For dealer Govind.

Cost price = Rs. 1000

Selling price = Rs. (1000 + 300) = Rs. 1300

For dealer Ankit,

Cost price = Rs. 1000

Profit = Rs. 300

Input tax credit = Rs. 120

Output tax $= Rs\left(\frac{12}{100} \times 1300\right)$ = Rs. 156

$\therefore$ Tax liability on dealer Govind = Rs. (156 – 120) = Rs. 36

4.2. i. Given quadratic equation is $3x^2 - 5x + 2 = 0$

On comparing it with $ax^2 + bx + c = 0,$ we get, $a = 3, b = -5$ and $c = 2$

Now, discriminant, $D = b^2 - 4ac = (-5)^2 - 4 \times 3 \times 2 = 25 - 24 = 1$

Since, $D > 0$, so the roots of the quadratic equation are real and distinct.

ii. Given quadratic equation is $x^2 - \frac{1}{3}x + \frac{3}{2} = 0$

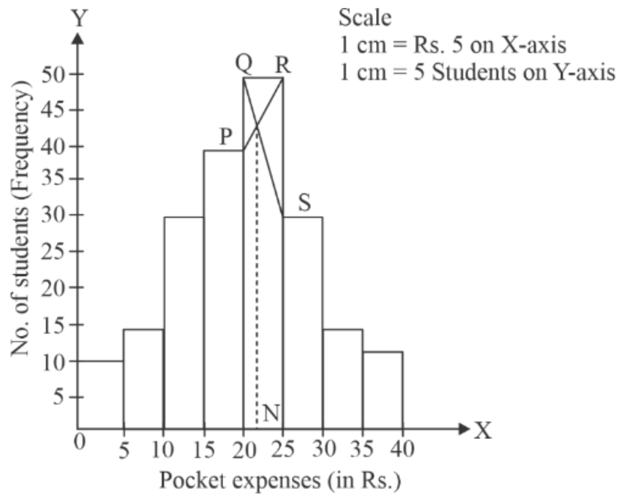
On comparing it with $ax^2 + bx + c = 0,$ we get

$a = 1, b = -\frac{1}{3}$ and $c = \frac{3}{2}$

Now, discriminant, $D = b^2 - 4ac = \left(\frac{-1}{3}\right)^2 - 4 \times 1 \times \left(\frac{3}{2}\right) = \frac{1}{9} - 6 = \frac{-53}{9}$

Since, $D < 1$, therefore the roots of the quadratic equation are imaginary.

4.3.



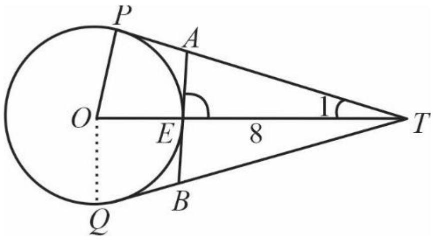
Question 5

5.1 $A=\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}, B=\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$ and $C=\begin{bmatrix}-3 & 2\\ -1 & 4\end{bmatrix}$

$=\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}+\begin{bmatrix}2 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}-3 & 2\\ -1 & 4\end{bmatrix}-5\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}$

$=\begin{bmatrix}4+0 & 2-2\\ 0+0 & 0+4\end{bmatrix}+\begin{bmatrix}-6-1 & 4+4\\ 0+2 & 0-8\end{bmatrix}-\begin{bmatrix}20 & 5\\ -15 & -10\end{bmatrix}=\begin{bmatrix}4 & 0\\ 0 & +4\end{bmatrix}+\begin{bmatrix}-7 & 8\\ 2 & -8\end{bmatrix}-\begin{bmatrix}20 & 5\\ -15 & -10\end{bmatrix}$

$=\begin{bmatrix}-3 & 8\\ 2 & -4\end{bmatrix}-\begin{bmatrix}20 & 5\\ -15 & -10\end{bmatrix}=\begin{bmatrix}-23 & 3\\ 17 & 6\end{bmatrix}$



5.2 **Given :** $r = OP = 5$ cm, $OT = 13$ cm

AB is tangent to the circle at E.

Construction : Join OQ

To find: Length of AB

Proof : $QT = PT$ ($\because$ tangents from ext. point T)

In ΔOPT and ΔOQT

$OP = OQ$ {radii of circle}

$OT = OT$ {Common}

$PT = QT$ {Length of tangent from point T}

$\therefore \Delta OPT \cong \Delta OQT$ {by SSS criteria}

$\therefore \angle PTO = \angle OTO$ {by C.P.C.T}

Now, OT or $OE \perp AB$ {$\because$ AB is tangent and OE or OT is passing through centre O}

$\because \angle AET = \angle BET = 90°$

Now, in ΔAET and ΔBET, we have

$\angle PTO = \angle QTO$ (proved)

$\angle AET = \angle BET = 90°$

$ET = ET$ (Common)

$\therefore \Delta AET \cong \Delta BET$ (by SAS) $\therefore AE = BE$ {by C.P.C.T}

Now, In ΔOPT and ΔAET

$\angle OPT = \angle AET = 90°$

$\angle 1 = \angle 1$ {common}

$\therefore \Delta OPT \sim \Delta AET$ {by AA similarity}

$$\frac{PT}{ET} = \frac{OP}{AE}$$

$$\frac{\sqrt{13^2-5^2}}{8}=\frac{5}{AE}$$

$$AE=\frac{40}{12}=\frac{10}{3}cm$$

$\therefore$ $AB = 2AE$

$AB = 2 \times 3.33$

$AB = 6.66$ cm

Hence, length of tangent AB = 6.66 cm

5.3. When $p(x)$ is divided by $(3x - 2)$, remainder $= p\left(\frac{2}{3}\right)$

Now, $= p\left(\frac{2}{3}\right)=9\times\left(\frac{2}{3}\right)^2-6\times\left(\frac{2}{3}\right)+2=9\times\frac{4}{9}-4+2= 4 - 4 + 2 = 2$

Question 6

6. Let the vertices of triangle be $A(3,4), B(-2,5)$ and $C(x_3, y_3)$

Let $Q(6, -1)$ be the centroid of triangle.

Centroid $=\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}$

$$b=\frac{3-2+x_3}{3}$$

$\Rightarrow$ $1+x_3=18$

$\therefore$ $x_3=17$

$$-1=\frac{4+5+y_3}{3}$$

$\Rightarrow$ $9+y_3=-3$

$\therefore$ $y_3=-12$

6.2 Hence, third vertex of triangle is $C(17,-12)$

(i) We know that $\sin\theta°=0$ and $\sin 90°=1$

$\therefore$ It is clear that $\sin\theta$ increase as θ increase

(ii) We know that $\cos\theta°=1$ and $\cos 90°=0$

$\therefore$ It is clear that as θ decreases, $\cos\theta$ increases

6.3 $S_n=5n^2-3n$

$S_1=a_1$

$=5(1)^2-3(1)=5-3=2$

$a_1=2$

$S_2=a_1+a_2$

$a_1 + a_2 = 5(1)^2 - 3(2) = 20 - 6 = 14$

$\therefore \quad a_1 + a_2 = 14$

$a_2 = 14 - 2$

$a_2 = 12$

Again, $S_3 = a_1 + a_2 + a_3$

$a_1 + a_2 + a_3 = 5(3)^2 - 3(3) = 45 - 9 = 36$; $a_1 + a_2 + a_3 = 36$

$14 + a_3 = 36$; $a_3 = 36 - 14$

$a_3 = 22$; $a_1 = 2,\ a_2 = 12,\ a_3 = 22$

Hence square becomes 2, 12, 22, …..

$a_1 = a = 2$

$d = 12 - 2 = 10$

$a_{16} = a + (16-1)d = 2 + 15 \times 10 = 2 + 150$

$a_{16} = 152$

Question 7

7.1. $3^{4x+1} - 2 \times 3^{2x+2} - 81 = 0$

$3^{4x} \cdot 3^1 - 2 \times 3^{2x} \times 3^2 - 81 = 0$

$\Rightarrow \quad \left(3^{2x}\right)^2 \times 3 - 2 \times 9 \times 3^{2x} - 81 = 0 \qquad \Rightarrow \quad 3\left(3^{2x}\right)^2 - 18\left(3^{2x}\right) - 81 = 0$

Let $3^{2x} = y$, then

$3y^2 - 18y - 81 = 0 \Rightarrow y^2 - 6y - 27 = 0$, $3y^2 - 18y - 81 = 0 \Rightarrow y^2 - 6y - 27 = 0$

$\Rightarrow \quad y^2 - 9y + 3y - 27 = 0 \quad \begin{Bmatrix} \because -27 = -9 \times 3 \\ -6 = -9 + 3 \end{Bmatrix}$

$\Rightarrow \quad y(y-9) + 3(y-9) = 0$

$\Rightarrow \quad (y-9)(y+3) = 0$

Either $y - 9 = 0$, then $y = 9$

or $y + 3 = 0$, then $y = -3$

If $y = 9$, then

$3^{2x} = 9 = 3^2$

Comparing

$2x = 2 \Rightarrow x = 1$

If $y = -3$, then

$3^{2x} = -3$ which has no real values

$\therefore \quad x = 1$

7.2.

Weight (C.I.)	Number of apples (f_i)	Mid alue (x_i)	$u_i = \frac{x_i - A}{h}$ A = 97.5, h = 5	$f_i u_i$
80-85	5	82.5	– 3	– 15
85-90	8	87.5	– 2	– 16
90-95	10	92.5	– 1	– 10
95-100	12	A = 97.5	0	0
100-105	8	102.5	1	8
105-110	4	107.5	2	8
110-115	3	112.5	3	9
Total	$\sum f_i = 50$			$\sum f_i u_i = -16$

Here, $\sum f_i = 50, \ \sum f_i u_i = -16, \ A = 97.5, \ h = 5$

Using step deviation method,

$$\text{Mean} = A + \frac{\sum f_i u_i}{\sum f_i} \times h = 97.5 + \frac{-16}{50} \times 5 = 97.5 - 16 = 95.9$$

Hence, the mean weight is 95.9 g,

Question 8

8.1. Total number of electric rod = 40

Out of 40, 6 has major defect

Sunita will accept ony (good + minor defect) electric bulb

i.e., favourable outcome for acceptable to Sunita = 30 + 4

(i) p(acceptatble to Sunita) $= \frac{34}{40} = \frac{17}{20}$

(ii) p(acceptable to trader) = Number of electric bulb with $\frac{\text{major defect}}{\text{Total electric bulb}} = \frac{6}{40} = \frac{3}{20}$

8.2. Radius of cone. $r = 2\,cm$ and height of cone, $h = 3\,cm$

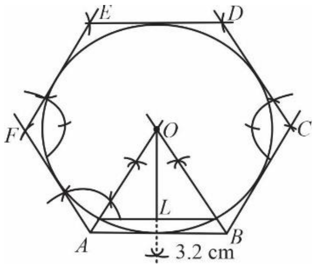
Then, volume of cone $= \frac{1}{3}\pi(r)^2 h = \frac{1}{3} \times \pi \times (2)^2 \times 3\,cm^3$

Also, radius of solid sphere, $R = 6\ cm$

Then, volume of solid sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (6)^3$

$$\text{Number of cones} = \frac{\text{Volume of solid sphere}}{\text{Volume of each cone}} = \frac{\frac{4}{3} \times \pi \times (6)^3}{\frac{1}{3} \times \pi \times (2)^2 \times 3} = 72$$

8.3. **(i)** Draw a line segment $AB = 3.2\ cm$



(ii) Draw rays at A and B making angle of 120° each and cut off $AF = BC = 3.2\ cm$.

(iii) Similarly at F and C, draw rays making angle of 120° each and cut off $FE = CD = 3.2\ cm$.

(iv) Join ED: $ABCDEF$ is a regular hexagon.

(v) Draw the angle bisector of $\angle A$ and $\angle B$ which intersect each other at O.

(vi) From O, draw $OL \perp AB$.

(vii) With centre O and radius OL draw a circle which will touch the sides of regular hexagon $ABCDEF$.

Question 9

9.1. We have, $\frac{4}{5}, \frac{3}{2}$ and $\frac{1}{10}$, which can be written as

$\frac{4\times2}{5\times2}, \frac{3\times5}{2\times5}$ and $\frac{1\times1}{10\times1}$ [$\because$ LCM of 5, 2 and 10 = 10]

$\Rightarrow \frac{8}{10}, \frac{15}{10}$ and $\frac{1}{10}$

Since, $1 < 8 < 15$ $\therefore$ $\frac{1}{10} < \frac{8}{10} < \frac{15}{10}$ [divide each term by 10]

$\Rightarrow \frac{1}{10} < \frac{4}{5} < \frac{3}{2}$

Hence, the given ratios in ascending order of their magnitude are $\frac{1}{10}, \frac{4}{5}$ and $\frac{3}{2}$.

Given, $PQ = 6cm$, $PR = 5.5cm$ and $\angle Q < 60°$.

9.2. **Steps of construction**

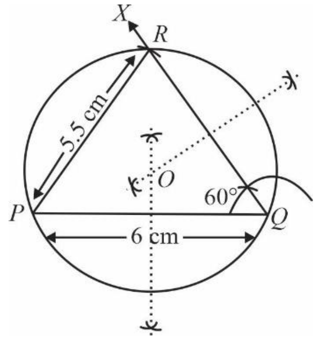
i. First, draw the base $PQ = 6cm$.

ii. Taking Q as centre, draw a ray QX making an angle of 60° with PQ.

iii. Now, taking P as centre, draw an arc of 5.5 cm, which cuts QX at point R.

iv. Join PR. Thus, PQR is the required triangle.

v. Draw the perpendicular bisectors of *PQ* and *QR*, which meet at point *O*.

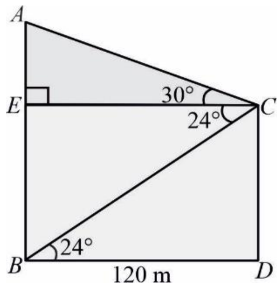


vi. Draw a circle with centre *O* and radius *OP*, which passes through all the vertices of ΔPQR.

Hence, it is the required circumcircle of ΔPQR.

9.3. Let the height of first tower be *AB* and second tower be *CD* and distance between both towers be *BD* i.e., *BD* = 120 *m* (given)

Since,



and $\angle ECB = \angle CBD$ {Given}

In ΔBDC, $\tan 24° = \dfrac{CD}{BD}$

$\Rightarrow \quad 0.4452 = \dfrac{CD}{120} \quad \Rightarrow \quad CD = 120 \times 0.4452 \quad \Rightarrow \quad CD = 53.424$

Hence, the length of second tower, $CD = 53.4\,m.$

In ΔAEC, $\tan 30° = \dfrac{AE}{EC}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{AE}{120} \quad [\because\ BD = EC = 120\,m]$

$\Rightarrow \quad AE = \dfrac{120}{\sqrt{3}} = 40\sqrt{3} = 40 \times 1.732 = 69.28\,m$

Hence, the length of first tower,

$AB = AE + BE = AE + CD \quad [\because\ CD = BE]$

$= 69.28 + 53.4 = 122.68\,m = 123\,m$

Question 10

10.1. $\frac{-17}{2} < \frac{-1}{2} - 4x \le \frac{15}{2}$; $\frac{-17}{2} < \frac{-1}{2} - 4x$; $4x < \frac{-1}{2} + \frac{17}{2}$

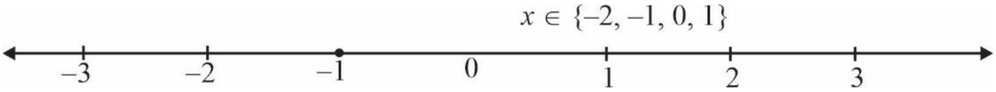
$4x < \frac{16}{2}$; $4x < 8$; $x < 2$

$x \in I$

$\frac{-1}{2} - 4x \le \frac{15}{2}$; $\frac{-1}{2}$ $\frac{-15}{2} \le 4x$; $\frac{-16}{2} \le 4x$

$-8 \le 4x$

$x \ge -2$



10.2. **(i)** Mean marks

$$(\bar{x}) = \frac{\text{sum of marks}}{\text{Number of student}} = \frac{11+19+7+13+18+21+9+5+20+17+16+21}{12}$$

$$= \frac{177}{12} \Rightarrow 14.75$$

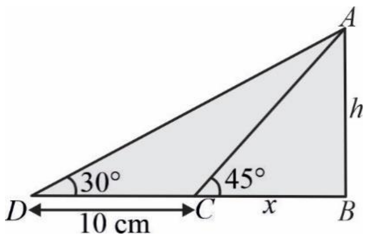
(ii) $\because$ We know that, if each observation are increased by a number 'p'.

Then, new mean $= \bar{x} + p$

So, here new mean marks $= \bar{x} + 3 = 14.75 + 3 = 17.75$

10.3. Let the height of the tower is h m.

In ΔABC, $\tan 45° = \frac{AB}{BC}$



$1 = \frac{h}{x}$

$x = h$ .....(i)

In $\Delta ADB, \tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{10+x}$

$10 + x = \sqrt{3}h$

$10 + h = \sqrt{3}h$ [from (i)]

$\sqrt{3}h - h = 10$

$h(\sqrt{3}-1) = 10$

$$h = \frac{10}{\sqrt{3}-1} = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{10 \times (\sqrt{3}+1)}{2} = 5 \times (\sqrt{3}+1)$$

$= 5(1.732+1) = 5 \times 2.732 = 13.66$ m